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IRE PROFESSIONAL GROUP ON AUTOMATIC CONTROL

The Professional Group on Automatic Control is an organization, within the framework of the IRE, of members with principal professional interest in Automatic Control. All members of the IRE are eligible for membership in the Group and will receive all Group publications upon payment of the prescribed fee.

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IRE TRANSACTIONS® on Automatic Control

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Control Concepts

NOT very long ago, the concept of automatic control was embodied in a simple diagram consisting of a block surrounded by a single line embellished with a summing point and direction arrows. Hundreds of papers have been written to explain the mysteries of stability and performance that pertain to this simple loop when actual physical equipment is substituted for the symbols representing it.

In spite of its simple appearance, the basic feedback concept has yielded a wide variety of mathematical problems. Extra loops have been added to the diagram, various switching modes have been investigated, and many graphical and mathematical techniques for solving the resulting mathematical equations have been developed. Actually, some papers pertaining to these developments have been essentially mathematical exercises. The feedback diagrams and transfer functions were manipulated into new forms, new mathematical interpretations, with little regard to practical applications. Sometimes it appeared as though the significance of the feedback concept was its ability to solve and even produce mathematical expressions. The basic purpose of feedback control was often obscured because the primary usefulness of feedback control is not its ability to generate a new class of mathematics, for mathematics has developed rather well without the use of feedback principles. Actually, the basic purpose and usefulness of feedback control is to make physical equipment more nearly resemble predictable mathematical models from which a successful performance must be determined and specified. In fact, if physical equipment used in control systems could be represented by invariant mathematical equations or transfer functions, there would be little use for feedback; if physical phenomena were mathematically predictable and measurable, disturbances and unwanted signals could be measured and cancelled with matching functions and phase reversals. However, physical equipment supplying useful energy does not have invariant mathematical characteristics. Gains, time constants, and linearity change with time and environment, with model and type. Drifts and other disturbances are not predictable or directly measurable; an invariant mathematical model cannot be realized, and accurate remote control cannot be achieved directly.

To minimize these physical variations and to make real equipment resemble mathematical models, feedback control was invented. It is feedback control which makes automatic control realizable and feasible. This is the principal usefulness of feedback control.

It may be noted that, whenever the feedback concept is the principal concern of a mathematical presentation, its significance may be judged by its consideration of practical realization, because, to the extent that the physical problems are neglected, the feedback concept and possibly the presentation itself may be disregarded.

In recent years, the concept of automatic control has been changing. More attention has been given to the control of large integrated systems involving complex computations, decisions, and logic.

The automatic control concept is no longer represented by the simple feedback control loop. In fact, it has become difficult to determine how a large control system should be represented, and it has become important to devise methods of obtaining satisfactory mathematical models of the system and its operations. Thus the concept of automatic control has been extended to many fields such as those of operations research, systems engineering, probability, statistics, game theory, queueing theory, information theory, and logic, all basically mathematical in nature.

Within the control systems are the computers, analog and digital, with the special mathematics and components needed to control the flow of information, to make the computations, and to produce decisions. It is evident that a mathematical model of the control system, a concise statement of its desired operation, is important. Feedback control may not be considered in the preliminary analysis; physical realizability may be of little concern. Actually, physical realizability is another problem of vital importance, and it is here that feedback control and all the theories that pertain to it become important. Again, it is the feedback concept that makes accurate automatic control realizable even in the most complex systems.

The automatic control field is also becoming more concerned with the concept of adaptive control. In a sense, this is a realization that even feedback systems cannot duplicate a mathematical model accurately and remain stable if the physical equipment and the input signal characteristics undergo drastic changes with time and environment.

To counteract unduly large changes in the physical equipment and to retain the stability of the desired closed-loop transfer function, it becomes necessary to vary characteristics of the physical equipment while it is in operation. This is done by a form of measurement and computation using a logic programming. Thus, in a large automatic control system, computers not only direct the feedback control loops, they change loop characteristics as needed. If correctly done, it may make component tolerances less stringent and physical realizability easier than it would be with simple feedback. Eventually, advanced adaptive systems may be able to develop their own adaptive program automatically. This would be truly an adaptive system in the biological sense, and this will be a development of another time. However, it appears that the fundamental significance of adaptive control is that it is an extension of the feed-

back control concept and that it will be needed primarily to obtain physical realizability of complex automatic control systems in the future.

There is still another major control concept, closely associated with adaptive control which has been referred to as optimizing or "optimalizing" control. This is closely associated with over-all system operation, as well as with physical realizability, because an optimalizing system has a computer which changes the system performance in an optimum manner from data obtained by measurement of a performance index. These systems have been developed to obtain optimum performance from engines and certain process controls, but significant problems remain as challenges for future development, especially in rapidly changing processes and environments.

Thus the automatic control concepts have developed into major sections: optimalizing, adaptive, and feedback control. All are expanding into new areas, all are important, all have new problems and applications. New contributions in any of these fields will be welcomed by these TRANSACTIONS, for it will be possible to extend the usefulness and the philosophy of these control concepts by circulating all worthwhile information.

—*The Editor*

Chairman's Report

This is a brief report on recent happenings which should be of interest to PGAC members. Of particular interest are the growing stature and responsibilities of the PGAC on a national and international scale.

CHANGES IN IRE POLICIES

During the past few months, the IRE Executive Committee and the Board of Directors have established a number of changes in IRE policy which directly affect the professional groups. Starting in 1959, the support given by the IRE to each group will be based on one-third of the cost of group publications, rather than on the basis of membership as it has in the past. This change has been made to promote publication activities by the groups. Another change has been made in respect to free distribution of the IRE NATIONAL CONVENTION RECORD and the IRE WESCON CONVENTION RECORD. In the past, PGAC members have received free of charge the portions of these two IRE CONVENTION RECORDS which contain the sessions sponsored by the PGAC. This free distribution to group members was at no cost to the PGAC and was supported entirely by sales of the IRE CONVENTION RECORDS at fairly high prices to non-Group members of the IRE, non-IRE members, and libraries. Because of increased costs, this publication plan is no longer feasible and free distribution must be discontinued. Instead, the IRE CONVENTION RECORD will be offered to Group members at a price considerably below that charged to non-Group members.

In view of these changes in policy, the PGAC has decided to publish the PGAC sessions at the IRE NATIONAL CONVENTION and WESCON as special PGAC TRANSACTIONS issues, which of course will be distributed to PGAC members free of charge. These issues will be in the normal IRE CONVENTION RECORD format and should be distributed within two months after each convention. The PGAC sponsored two sessions of five papers at the Convention and cosponsored with the PGEC and the PGIT a session on Theory and Practice in Russian Technology.

Another IRE action of particular importance to the PGAC is the decision by the Board of Directors that the IRE should affiliate with and support international organizations of interest to the professional groups. For the past two years, the PGAC has actively supported and paid dues to the American Automatic Control Council which is the United States representative organization in the International Federation on Automatic Control. The PGAC was thus the first IRE Professional Group to affiliate with and support an international organization. The Board of Directors has now decided that in instances such as this, the IRE itself should affiliate with and support the organization, rather than the professional group. The IRE has thus taken over support of the AACC from the PGAC, starting in 1959. As part of this change in policy, the Board of Directors has given the Professional Groups complete responsibility for representing the IRE, and for organizational details in connection with international meetings, publications, and participation in committees. The PGAC Administrative Committee recognizes the nature of this responsibility, and will strive to represent the best interests of the IRE and the PGAC in relations with the AACC and the IFAC.

JOINT CONTROL MEETINGS

When the PGAC scheduled its first national control meeting to be held in Dallas, November 4–6, 1959, and

invited other professional societies to participate, it was hoped that this meeting could be a start toward joint control meetings instead of separately-sponsored meetings each year by the separate societies. This hope has been fulfilled, and the AIEE, the ISA, the ASME, the IRE PGIE, and the Dallas section of the IRE have all agreed to participate in the Dallas meeting. The AIEE has also scheduled a Components Conference for November 5 and 6 which will be held in conjunction with the National Control Conference. PGAC members should be interested to learn that this start on co-operative control meetings has recently been formalized by a decision of the American Automatic Control Council to recognize one national control meeting per year. It has been agreed by the AACC sponsor societies that these national control meetings will be sponsored each year by a different society with other societies on a participating basis, and that the meetings will start with the PGAC National Control Conference in Dallas. Meetings in successive years will be sponsored by the ASME, the ISA, the AIEE, and the AICHE. The PGAC is in accord with this action by the AACC and has accepted AACC recognition of the Dallas Conference. It is felt that this banding together of the interested groups in the various societies to sponsor one national control meeting per year is a significant step in the control field, and one that should have large benefits in the years to come.

PGAC Growth

The PGAC has continued to grow since its inception four years ago and currently has 2993 paid members, 240 student members, 9 affiliate members from societies other than the IRE, and 107 unpaid members. The PGAC is thus the ninth largest of the 26 professional groups. The PGAC also has 10 active chapters. Although the PGAC is active on a national scale with its sponsorship of sessions at the IRE National Convention and WESCON, the sponsorship of the 1959 National Control Conference, and its publication of TRANSACTIONS, much of the strength of the PGAC is in its chapter activities. For this reason, the PGAC is anxious to promote the formation of additional chapters where group membership is large enough to warrant such formation. Areas where chapter formation appears possible are:

	Members		*Members*
Chicago	97	Pittsburgh	41
Cleveland	25	St. Louis	46
Detroit	61	San Diego	39
Indianapolis	31	San Francisco	153
Milwaukee	34	Seattle	36
New York	208	Tokyo	48
Northern New Jersey	116	Washington	77

PGAC members in these and other sections who are interested in organizing chapters are urged to contact the PGAC Chapter Chairman, Robert Wilcox, Sylvania Electric Products Inc., 100 First Avenue, Waltham 54, Mass.

—JOHN E. WARD

The Issue in Brief

Special features of this issue include a discussion of the American Automatic Control Council by its first chairman, the first section of a three-part tutorial paper, and two annotated bibliographies.

The American Automatic Control Council, Rufus Oldenburger

The AACC is an organization which represents the automatic control interests of North America in IFAC, the International Federation of Automatic Control. The IRE, an affiliate member of AACC, is represented by the PGAC, and therefore, it should be of particular interest to PGAC members. We are fortunate in having Dr. Oldenburger, the first chairman of AACC, describe the functions and importance of the organization. In a future issue, it is planned to have a similar article on IFAC and the world organization of which it is comprised.

Comparison of Lead Network, Tachometer, and Damper Stabilization for Electric Servos, George A. Biernson

The advantages and disadvantages of three common stabilization techniques are discussed in terms of bandwidth, velocity constant, torque constant, transient response, backlash, noise, and amplifier gain. The purpose of the comparison is to provide a basis for selection of the most appropriate compensation for a particular application. A simple, but powerful method of analysis involving the asymptotic crossover frequency of the servo is used to describe and compare servo performance using the various methods of compensation.

The Analysis of Sampled-Data Control Systems with a Periodically Time Varying Sampling Rate, Eliahu I. Jury and Francis J. Mullin

The z transform is used to determine the response of sampled-data systems which have a repetitive sampling pattern with a varying time between individual samples. Such systems are found in time-sharing computers or in telemetering devices where information is not available in equal time intervals. These systems may be described by linear difference equations with periodic coefficients. However, at the sampling instants, the difference equation may be considered linear with constant coefficients. By considering a difference equation for each sampling instant in a sampling period, a series of difference equations may be formed and then solved using the z transform. The response between sampling instants can also be found using solutions of these difference equations.

Automatic Control of Vector Quantities, Allen S. Lange

This is the first long tutorial article to be featured in these TRANSACTIONS. It has taken considerable time and perserverance to prepare, and it will be given in three consecutive parts. It concerns mathematical techniques using matrix notations which are applicable to the design of automatic control systems involving coordinate converters, guidance computers, and stable platforms with inputs, outputs, and disturbances characterized as vector quantities. It is shown that vector matrix elements represent vector cartesian components in three-dimensional space which can be measured or generated by combinations of commercially available instruments. Thus, techniques are developed to form a bridge between classical mechanic theories and automatic control technology necessary for modern weapons system design.

Interestingly enough, the next two papers use matrix notation in solving different control problems.

On the Synthesis of Feedback Systems with Open-Loop Constraints, John A. Aseltine

The problem of synthesizing control systems where some of the open- and closed-loop roots are both specified is discussed, and a method of solving the problem is presented. It is based on an inverse root-locus plot and algebraic equations pertaining to the open-loop pole and zero locations. Examples are given for systems through the fourth order in which resulting linear algebraic equations are readily solved for the required poles and zeros of the compensating devices.

Complex Curve Fitting, Ezra C. Levy

It is often required that the frequency response of a system, determined by experimental tests, be fitted by an algebraic expression, usually a ratio of two frequency dependent polynomials. A method of evaluating the polynomial coefficients is presented. It is based on the miniaturization of the weighted sum of the squares of the errors between the absolute magnitudes of the actual function and the polynomial ratio for various frequencies, and the problem is solved by evaluating determinants involving functions of system amplitude ratio and phase shift. This form is particularly adaptable to digital computing methods because of the simplicity in the required programming.

Characteristics of the Human Operator in Simple Manual Control Systems, Jerome I. Elkind and Carma D. Forgie

Systems with manual control have a unique property in that the human operator can modify his own characteristics in an attempt to match the requirements of the control situation. This property is investigated in this paper by measuring the characteristics of two different manual control systems with a family of gaussian input signals with power-density spectra having several shapes, bandwidths, and center frequencies. The experimental results in the form of graphs show how the human operator characteristics depend upon input-signal characteristics. Simple analytic models that approximate the measured results are derived for both systems.

It should be noted that the analytic models of the human operator contain a transportation lag which is referenced extensively in the succeeding article.

Transportation Lag—An Annotated Bibliography, Robert Weiss

In addition to the human operator characteristics derived in the preceding paper, there are many other functions in control systems which contain transportation lags. They are encountered in process control, thermal systems, including nuclear reactors, rocket motors, traveling waves, magnetic amplifiers, high-speed aerodynamic control, and economic control. This is a bibliography which includes abstracts of a number of references concerning the problems peculiar to transportation lag in control systems.

Adaptive or Self-Optimizing Control Systems—A Bibliography, Peter R. Stromer

In the December, 1958, issue of these TRANSACTIONS, there was an excellent survey paper of adaptive control systems.[1] It also contained a substantial bibliography of the subject. Under circumstances similar to those pertaining to his last contribution to these TRANSACTIONS,[2] P. R. Stromer has submitted a more detailed bibliography which duplicates some of the previous material, but quite appropriately because it contains a short abstract of each reference. Bibliographies have been well received in the past issues, and the two bibliographies in this issue should add appreciably to the past reference material.

Correspondence

A discussion is given by Daniel Levine about stability diagrams on magnitude angle charts instead of on polar coordinates.

PGAC News

Announcements of interesting control meetings and awards are presented.

PGAC Membership Directory

A customary feature of the Spring issue of these TRANSACTIONS is the membership directory, and it is revised in this issue. It is interesting to note that there are members of the PGAC in all parts of the world.

[1] J. A. Aseltine, A. R. Mancini, and C. W. Sarture, "A survey of adaptive control systems," IRE TRANS. ON AUTOMATIC CONTROL, No. PGAC-6, pp. 102–108; December, 1958.

[2] P. R. Stromer, "A selective bibliography on sampled-data systems," IRE TRANS. ON AUTOMATIC CONTROL, No. PGAC-6, pp. 112–114; December, 1958.

The American Automatic Control Council*

RUFUS OLDENBURGER†

An international meeting of automatic control was held in Heidelberg, Germany, September 25–29, 1956, under the auspices of the joint control Committee of the German electrical and mechanical engineering societies. This committee is known as the VDI/VDE—Fachgruppe Regelungstechnik. Other international meetings on automatic control had been held previously. On July 16–21, 1951, one took place in Cranfield, England. This was called the "Conference on Automatic Control." It was followed by the "Frequency Response Symposium" in New York City, December 1–2, 1953. The Heidelberg meetings were such a success that a demand arose for an international federation which could assist the host country in the matter of publicity, securing papers, and other aspects of the organization of future international control meetings.

On September 27, 1956, a meeting was held at the University of Heidelberg attended by:

Otto Grebe, Germany	J. M. L. Jansen, Netherlands
G. Ruppel, Germany	J. G. Balchen, Norway
G. Muller, Germany	J. R. Jensen, Denmark
H. Kindler, Germany	P. J. Nowacki, Poland
W. Pohlenz, Germany	G. Evangelisti, Italy
Rufus Oldenburger, U.S.A.	J. Boas Popper, Israel
A. Tustin, Great Britain	Ph. Passau, Belgium
J. F. Coales, Great Britain	V. Broida, France
Harold Chestnut, U.S.A.	M. Ajnbinder, Belgium
J. H. Westcott, Great Britain	L. V. Hamos, Sweden
A. M. Letov, U.S.S.R.	S. Vladimir, CSR (Czechoslovakia)
H. Marzendorfer, Austria	
H. Mesarović, Yugoslavia	B. Hanus, CSR

K. Izawa, Japan

The following resolution was adopted and signed by all present:

"The undersigned favor the founding of an international federation of automatic control and declare themselves prepared to work in their respective countries for the organization of such a union. This federation is to have the following objectives:

1) To facilitate the interchange of information in automatic control and to promote progress in this field.
2) To organize international congresses in automatic control."

After adoption of this resolution a provisional committee was formed to promote the organization of the federation, composed of:

Broida (France), Chairman	Nowacki (Poland)
Grebe (Germany)	Oldenburger (U.S.A.)
Letov (U.S.S.R.)	Welbourn (United Kingdom)

Ruppel (Germany), Secretary

* Manuscript received by the PGAC, February 19, 1959.
† Purdue University, Lafayette, Ind.

Subsequently, a U.S.A. control committee was formed by the societies listed bleow, with the following delegates and alternates:

American Institute of Electrical Engineers
- Delegate, Harold Chestnut
- Alternate, Gerhart Heumann

American Society of Mechanical Engineers
- Delegate, Rufus Oldenburger
- Alternate, William E. Vannah

Institute of Radio Engineers
- Delegate, John C. Lozier
- Alternate, Eugene M. Grabbe

Instrument Society of America
- Delegate, Robert Jeffries
- Alternate, John Johnston, Jr.

American Institute of Chemical Engineers
- Delegate, Joel O. Hougen
- Alternate, Norman H. Ceaglske

This committee was eventually named "The American Automatic Control Council," abbreviated AACC or A^2C^2. It adopted a constitution which states that the aims of the Council are:

"1) To promote cooperation among the various technological societies in the United States or their Divisions and Committees which are devoted to or have an active interest in the theory and practice of control engineering.
2) To represent the U.S.A. in the programs and activities of the International Federation for Automatic Control.
3) To provide representatives in the affairs of the International Federation of Automatic Control who will most faithfully reflect the cross section of technical opinion and society policy existing in the U.S.A."

To achieve these objectives, the functions of the Council are:

"1) To act as an advisory and coordinating agency concerning international matters of mutual interest to the constituent societies of the Council.
2) To represent the constituent societies of the Council on matters of an international nature coming before the International Federation for Automatic Control.
3) To expedite and administer on behalf of the constituent societies those activities authorized by the Council.
4) To restrict its National activities to the encouragement of cooperation among U. S. technical and scientific organizations and assume no juris-

diction in connection with the facilities, meetings, activities or policies of these organizations."

Regarding membership in the Council, the constitution states that an organization may become a constituent society of the American Automatic Control Council upon meeting the following qualifications:

"1) The organization is a professional, scientific or engineering society of individuals in which a division or group is actively engaged in Control engineering.
2) The organization's request for membership in AACC receives the affirmative vote of not less than two thirds of the delegates representing the constituent societies of the Council.
3) The organization has its headquarters located within the continental limits of the U.S.A."

Each Council member is represented by one delegate and one alternate. The Council is supported by annual dues paid by its members. The Council also adopted a set of bylaws detailing its mode of operation.

A provisional draft of the constitution of the international federation was prepared by the Council for presentation to the provisional committee, which met in Düsseldorf, Germany, April 25–27, 1957. At this meeting, the constitution prepared by the American Council was approved with modifications, and definite plans were made for the organizational meeting of the Federation. This meeting was held in Paris, September 11 and 12, 1957. It was attended by delegates from Austria, Belgium, China, France, Germany, Hungary, Italy, Japan, Netherlands, Norway, Poland, Sweden, Switzerland, Turkey, United Kingdom, U.S.A., U.S.S.R., and Yugoslavia. The meeting was held in the École des Arts et Métiers. The federation was officially formed after adoption of the constitution. Harold Chestnut was elected president, A. M. Letov, first vice president, V. Broida, second vice president, G. Ruppel, secretary, and G. Lehmann, treasurer. These men and six others were elected to the Executive Council of the federation. The name for the organization was chosen to be "The International Federation of Automatic Control," abbreviated IFAC. An invitation to hold the first IFAC congress in Moscow in 1960 was accepted.

The constitution of IFAC states:

"1) IFAC is to promote among nations the science of automatic control.
2) Automatic Control is deemed to cover the field of open and closed loop (feedback) control of physical systems (including servomechanisms, instrumentation, data processing and computers when part of control systems) in theoretical and applied aspects."

The constitution states that to achieve these aims, the following methods are to be employed:

"1) IFAC is to sponsor international congresses.
2) IFAC is to promote the interchange and circulation of information on automatic control activities in co-operation with existing national and international organizations.
3) IFAC is to take such other steps as may be considered necessary to promote the aim of the Federation, *e.g.*, to print, publish or distribute the proceedings or reports of IFAC or any other relevant papers."

Regarding membership, the constitution contains the following:

"1) For each country one scientific or professional engineering organization or one council formed by two or more such organizations engaged in automatic control activity is entitled to become a member of IFAC.
2) Each application for membership of IFAC shall be addressed to the Executive Council. If the latter decides that the application corresponds with the regulations of Article 7 (the last item 1 above), the Executive Council will then submit the application for membership to the General Assembly which will authorize the admission on a simple majority of votes cast. Admission includes the obligation to recognize and adhere to this Constitutuion.
3) Membership may be terminated by a vote of the General Assembly if a member is one year in arrears in payment of its subscription, or by a declaration of a member organization which is to be submitted to the President or to the Secretary prior to July 1st in order to become effective at the end of the current year."

The General Assembly of IFAC is its supreme body. English is the official language for IFAC decisions, but French, German, and Russian are also official. The management of IFAC is vested in the Executive Council. Members pay annual dues to IFAC for its support.

Since the formation of the Federation, the AACC has been active in promoting the reduction of national control conferences of the member societies to one joint conference a year, and in increasing technical emphasis on control activities. Its principal function, however, has been to carry on negotiations in the control area with organizations in other countries. The Council has arranged exchange visits between control experts of the United States and the U.S.S.R., and the exchange of journals between the U.S.A. and other countries. The visit of Professors A. A. Voronov, A. B. Chelyuskin, and V. D. Pavlov, April 1–8 to the ASME-IRD meetings at the University of Delaware was under the auspices of the Council, as well as the return visit of N. Cohn, E. P. Epler, J. Felker, E. M. Grabbe, J. W. Herwald, E. J. Kelly, R. T. Kochenburger, H. W. Mergler, G. C. Newton, R. L. Palmer, P. S. Sprague, W. E. Vannah, and H. W. Ziebolz, August 17 to September 2 to the U.S.S.R.

The visit of Professors and Engineers A. M. Letov, A. B. Chelyuskin, B. Naumov, A. Petrovsky, P. Sypchek, and A. Ignatiev, September 14–29 to the first meetings in Philadelphia, Pa., New York City, N. Y., Boston, Mass., and Chicago, Ill. was also under the auspices of the Council.

The Council is actively engaged in securing papers for the Moscow congress scheduled for June, 1960. John Lozier has over-all responsibility for American participation in this congress. E. M. Grabbe is coordinator of papers, whereas Prof. John Truxal, N. B. Nichols, and David M. Boyd, are chairmen of committees to secure papers on theory, instrumentation, and applications respectively. Dean John R. Ragazzini is AACC editor of the English version of the Moscow proceedings.

Prof. D. P. Eckman, Mark Princi, and Prof. T. J. Higgins are AACC representatives on IFAC technical advisory, nomenclature, and bibliography committees, respectively. Prof. Eckman is chairman of the IEAC committee to which he belongs.

AACC will be host to the first general assembly of IFAC, at which new officers will be elected, and at which the constitution may be amended. This assembly will take place in Chicago during September 16–18, 1959, just before the annual meetings of the Instrument Society of America in which some foreign IFAC delegates will participate. David Boyd, Albert Sperry, Donald Bergman, Prof. R. W. Jones, and W. F. Stevens are on the local AACC committee of arrangements for this meeting.

The activities of AACC are expanding constantly. This is being done by working through the member societies where possible, and drawing on personnel from these societies. New societies are expected to join AACC to make it even a more effective instrument in promoting the automatic control field.

Comparison of Lead Network, Tachometer, and Damper Stabilization for Electric Servos*

GEORGE A. BIERNSON†

Summary—**Three types of compensation widely used to achieve stable operation in instrument servomechanisms are: the lead network, tachometer feedback, and the viscous-coupled-inertia damper. The paper compares these types of compensation in such matters as servo bandwidth, velocity constant, torque constant, transient response, tolerance to gear-train backlash, noise, and required amplifier gain. The purpose of this comparison is to provide a basis for selection of the most appropriate type of compensation for a particular application.**

The paper also serves to illustrate a method of analyzing servo performance in terms of the asymptotic gain-crossover frequency. Although this method may be theoretically trivial, it actually is a powerful tool for analyzing the performance of feedback control systems.

Introduction

THIS PAPER compares three basic types of compensation used for achieving a wide bandwidth in electric servomechanisms, namely, lead network, tachometer feedback, and viscous-coupled-inertia damper. They represent, respectively, examples of the more general compensation techniques: cascade compensation, feedback compensation, and load compensation.

In order to make this comparison, it is important to have a clear definition for the concept of bandwidth. The criterion to be used for the bandwidth of a feedback-control loop is the gain crossover frequency, designated ω_{cg} and defined as the frequency where the loop gain is unity. The loop gain is the magnitude of the loop frequency response $G(j\omega)$ [sometimes called $KG(j\omega)$]. The justification for using the gain crossover as a measure of bandwidth is that it defines quite accurately the rise time of the step response for adequately stable feedback-control systems; and also it gives a reasonable close approximation for the maximum value of the error response to a unit ramp [3]. These relations are discussed in more detail later.

A useful frequency parameter related to the gain-crossover frequency is the asymptote-crossover frequency ω_c. It is defined for feedback-control loops in which the loop gain approaches, in the region near gain crossover, an asymptote with a slope inversely proportional to frequency (*i.e.*, -10 decilogs per decade or -20 decibels per decade). In practical systems, this condition is almost always met. The frequency where that asymptote, extended if necessary, crosses the unity-gain axis is called the asymptote-crossover frequency ω_c. Generally, this frequency is reasonably close to the actual gain-crossover frequency ω_{cg}, and is much easier to calculate. In this paper, the more convenient parameter ω_c is used as a basis for comparing bandwidths.

* Revised manuscript received by PGAC, January 23, 1959. The material of this paper was first presented as "Comparison of Lead Network, Tachometer, and Damper Compensation for Instrument Servos," Servomechanisms Lab., Mass. Inst. Tech., Cambridge, Tech. Mem. 7138-TM-10; December, 1956.

† Sylvania Electric Products, Waltham, Mass.

Compensation Components

Fig. 1(a) shows a lead network. The transfer function is

$$\frac{s+\omega_l}{s+\alpha\omega_l} \tag{1}$$

where the attenuation ratio α is

$$\alpha = (R_1 + R_2)/R_2 \tag{2}$$

and the lead-network break-frequency is

$$\omega_l = 1/R_1C_1. \tag{3}$$

A tachometer is a generator mounted on the motor shaft. It delivers a voltage proportional to the tachometer speed and hence also proportional to the motor speed.

A viscous-coupled inertia damper is illustrated in Fig. 1(b). The damper consists of a heavy slug (inertia J_d) coupled to the motor shaft (inertia J_m) by viscous damping (damping factor f_d). For high-frequency oscillations of the motor shaft, the damper slug remains stationary, and the oscillations are damped out by the viscous damping f_d between the motor and the slug. For constant velocities of the motor, however, the damper slug picks up speed and eventually moves with the same speed as the motor. Thus, for constant velocities, there is no slip between the motor shaft and the damper and hence no power loss.

The transformed equations for the motor-damper combination shown in Fig. 1(b) are

$$T_m = \Omega_m(J_m s + f_m) + f_d(\Omega_m - \Omega_d) \tag{4}$$

$$f_d(\Omega_m - \Omega_d) = J_d s \Omega_d \tag{5}$$

where

T_m = motor torque
J_m = motor-shaft inertia
f_m = motor-shaft damping factor
f_d = damper-damping factor
J_d = damper-slug inertia
Ω_m = motor-angular velocity
Ω_d = damper-slug angular velocity.

The resultant equation relating motor velocity to motor torque is

$$\frac{\Omega_m}{T_m} = \frac{s+\omega_d}{J_m[s^2 + s(\omega_m + \alpha\omega_d) + \omega_m\omega_d]} \tag{6}$$

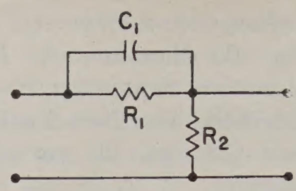
where

$$\omega_m = f_m/J_m = \text{motor-break frequency} \tag{7}$$

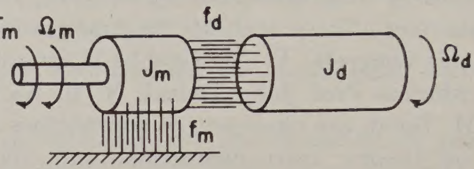
$$\omega_d = f_d/J_d = \text{damper-break frequency} \tag{8}$$

$$\alpha = (J_m + J_d)/J_m = \text{inertia ratio.} \tag{9}$$

In practice, ω_m is practically negligible with respect to $\alpha\omega_d$, so that (6) can be approximated quite accurately by replacing ω_m by the even more negligible term ω_m/α:



(a)



(b)

Fig. 1—Compensation devices. (a) Lead network. (b) Motor and damper.

$$\frac{\Omega_m}{T_m} = \frac{s+\omega_d}{J_m\{s^2 + s[(\omega_m/\alpha) + \alpha\omega_d] + \omega_m\omega_d\}}. \tag{10}$$

This can be factored to give

$$\frac{\Omega_m}{T_m} = \frac{s+\omega_d}{J_m(s+\omega_m/\alpha)(s+\alpha\omega_d)}. \tag{11}$$

Examination of Systems

Block diagrams of lead network, tachometer, and damper systems are shown in Fig. 2. The parameters and variables shown are

θ_i = input angle
θ_0 = output angle of load
θ_e = error in angle
K_a = amplifier gain including synchro gain
K_m = motor gain (torque per-unit voltage)
V_m = voltage applied to motor
T_m = motor-shaft torque
Ω_m = motor-shaft angular velocity
θ_m = motor-shaft angle
n = gear reduction, motor to load
K_t = tachometer gain, which may be partly produced by amplifier.

Lead-Network System

The loop transfer-function for the lead-network system in Fig. 2(a) is

$$G = \frac{K_aK_m}{J_m n}\,\frac{(s+\omega_l)}{s(s+\omega_m)(s+\alpha\omega_l)}. \tag{12}$$

As will be seen by the following analysis, it is very convenient to express this transfer function in terms of the asymptotic gain crossover frequency. This can be done quite simply by rewriting (12) in the form:

$$G = \frac{K(s+\omega_l)}{s(s+\omega_m)\left(\dfrac{s}{\alpha\omega_l}+1\right)} \tag{13}$$

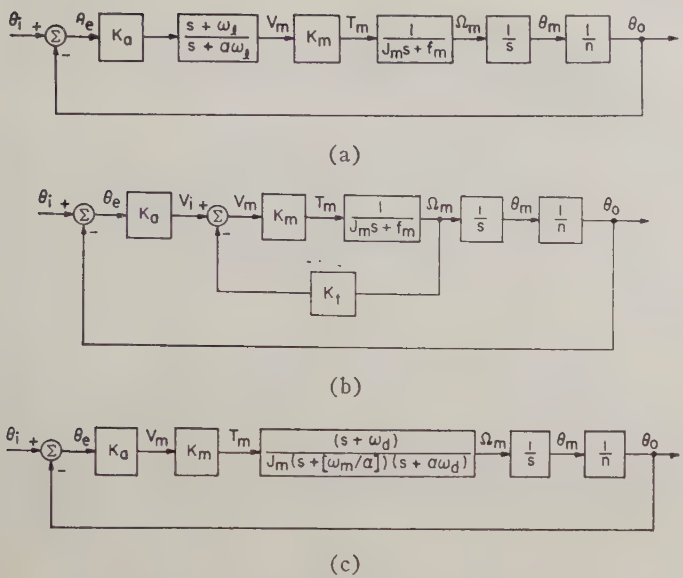


Fig. 2—System block diagrams. (a) Lead network system. (b) Tachometer system. (c) Damper system.

where K is a constant. In (13), all of the factors with break frequencies below ω_c in frequency are written in the form $(s+\omega_x)$; while those with break frequencies above ω_c are written in the form $[(s/\omega_x)+1]$. Consequently, in the frequency region near ω_c, (13) approximates K/s and its magnitude approaches the asymptote K/ω. Since this asymptote is unity when ω is equal to K, then K must by definition be equal to the asymptote-crossover frequency ω_c. Thus (13) can be expressed as

$$G = \frac{\omega_c(s+\omega_l)}{s(s+\omega_m)\left(\dfrac{s}{\alpha\omega_l}+1\right)}. \tag{14}$$

Setting (12) equal to (13) gives the following expression for ω_c:

$$\omega_c = \frac{K_aK_m/n}{J_m\alpha\omega_l}. \tag{15}$$

Writing the loop transfer function in terms of ω_c as is done in (14) is unusual and so may seem rather arbitrary. In the customary method, all the factors of the transfer function are expressed in the form $(\tau s+1)$, and the constant that results represents the low-frequency slope of the frequency-response magnitude plot (which is the positional, velocity, or acceleration constant). Although this customary method is almost universally employed, like the author's method, it is merely a convention, and either convention is really as arbitrary as the other. The important consideration is which convention expresses the transfer function in the most convenient form for engineering evaluation.

The important advantage of the author's convention is that it yields the asymptotic gain-crossover frequency as the gain constant in the transfer function expression. This parameter generally is much more directly related to system performance than is the low-frequency slope of the loop transfer function. From a theoretical point of view, this change of convention is trivial, but nevertheless, the new convention represents a very powerful analytical tool. Its potentialities can best be expressed by showing how it is applied to specific examples, as is done in the remainder of the paper.

The velocity constant K_v is defined as

$$K_v = sG\big|_{s=0}. \tag{16}$$

Substituting (14) into (16) gives for K_v,

$$K_v = \left(\frac{\omega_l}{\omega_m}\right)\omega_c. \tag{17}$$

The torque constant K_T of a servomechanism relates a steady load torque applied at the output shaft to the resultant error. The block diagram of Fig. 2(a) shows that the dc gain between the error θ_e and the motor torque T_m is

$$\frac{T_m}{\theta_e} = \frac{K_aK_m}{\alpha}. \tag{18}$$

Since the torque at the output shaft is n times the torque at the motor, the torque constant is n times this ratio, or

$$K_T = \frac{nK_aK_m}{\alpha}. \tag{19}$$

In terms of ω_c, this is equal to

$$K_T = n^2J_m\omega_c\omega_l. \tag{20}$$

It should be noted that n^2J_m is the motor-plus-load inertia reflected to the output shaft, where the torque constant is measured. Thus the torque constant can also be expressed as $(J_0\omega_c\omega_l)$ where J_0 is the inertia reflected to the output.

Tachometer System

The closed-loop transfer function Ω_m/V_i of the tachometer loop in Fig. 2(b) can readily be shown to be equal to

$$\frac{\Omega_m}{V_i} = \frac{K_m}{J_ms+(f_m+K_tK_m)}. \tag{21}$$

Comparing this expression with the transfer function Ω_m/V_m of the motor alone shows that the tachometer signal effectively increases the motor damping f_m by the amount K_tK_m, which may be considered to represent a tachometer damping factor f_t:

$$f_t = K_tK_m. \tag{22}$$

This result may be justified in physical terms as follows. The tachometer signal effectively produces a torque on the motor proportional to tachometer speed, and therefore proportional to motor speed; and this torque opposes the motor velocity. Thus, this torque acts like a viscous drag on the motor. The damping factor f_t in

(22) is the ratio of this tachometer-signal torque to the tachometer (or motor) velocity.

Although the tachometer signal has the effect of a viscous drag in terms of linear considerations, the subtraction of the tachometer signal is performed prior to the motor, rather than directly on the motor shaft; and hence, tachometer feedback does not reduce the power capabilities of the motor, as does an actual viscous drag.

The ratio f_t/J_m can be considered to represent a tachometer-damping break frequency ω_t:

$$\omega_t = f_t/J_m. \tag{23}$$

The transfer function for the position loop of the tachometer system in Fig. 2(b) can be expressed in terms of ω_t as follows:

$$G = \frac{K_a/n}{K_t\left[1 + \dfrac{\omega_m}{\omega_t}\right]} \frac{(\omega_t + \omega_m)}{s[s + (\omega_t + \omega_m)]}. \tag{24}$$

An asymptotic magnitude plot of G is shown in Fig. 3(b). The figure shows that G is expressed in terms of ω_c by

$$G = \frac{\omega_c}{s\left[\dfrac{s}{\omega_t + \omega_m} + 1\right]}. \tag{25}$$

Comparing (24) and (25) shows that ω_c is equal to

$$\omega_c = \frac{K_a/n}{K_t\left[1 + \dfrac{\omega_m}{\omega_t}\right]}. \tag{26}$$

Applying (16) to (25) gives for the velocity constant

$$K_V = \omega_c. \tag{27}$$

When only a constant load torque is applied, there is no tachometer feedback signal, and therefore the tachometer feedback signal can be disregarded in calculating the torque constant. Since the dc gain between θ_e and T_m (disregarding the tachometer feedback signal) is K_aK_m, the torque constant is n times this or

$$K_T = nK_aK_m. \tag{28}$$

In terms of ω_c this is

$$K_T = n^2(f_t + f_m)\omega_c = n^2J_m(\omega_t + \omega_m)\omega_c. \tag{29}$$

Damper System

The loop transfer-function of the damper system in Fig. 2(c) is

$$G = \frac{K_aK_m/n}{J_m\alpha\omega_d} \frac{(s + \omega_d)}{s[s + (\omega_m/\alpha)][(s/\alpha\omega_d) + 1]}. \tag{30}$$

An asymptotic magnitude plot of this is shown in Fig. 3(c). From this, it can be seen that G may be expressed as follows in terms of ω_c;

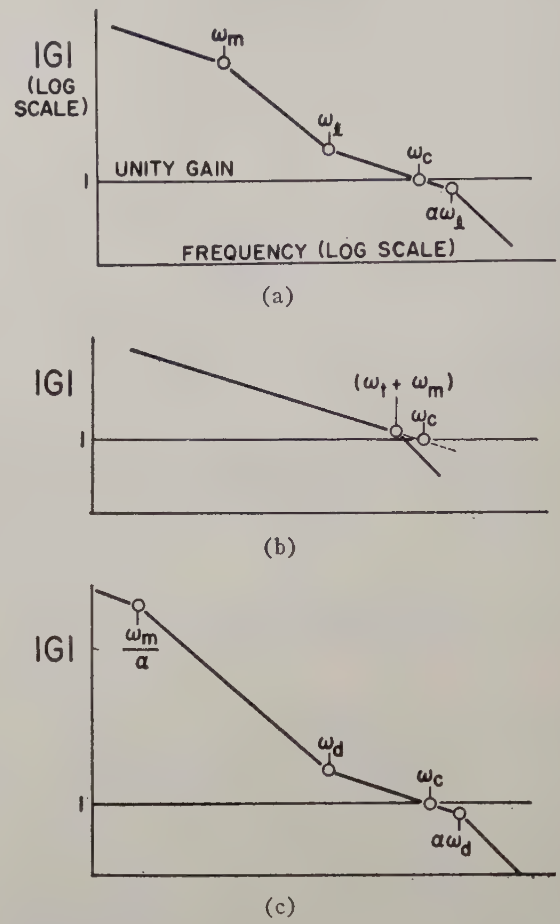


Fig. 3—Frequency responses. (a) Lead network system. (b) Tachometer system. (c) Damper system.

$$G = \frac{\omega_c(s + \omega_d)}{s[s + (\omega_m/\alpha)][(s/\alpha\omega_d) + 1]}. \tag{31}$$

Setting (30) and (31) equal gives for ω_c,

$$\omega_c = \frac{K/K_m/n}{J_m\alpha\omega_d}. \tag{32}$$

Applying (16) to (31) gives for the velocity constant

$$K_V = \omega_c \frac{\alpha\omega_d}{\omega_m}. \tag{33}$$

In Fig. 2(c) the dc gain between θ_e and T_m is K_aK_m, so that the torque constant is

$$K_T = nK_aK_m. \tag{34}$$

In terms of ω_c this is

$$K_T = n^2J_m\alpha\omega_d\omega_c = n^2(J_m + J_d)\omega_d\omega_c. \tag{35}$$

This analysis relates only to the viscous-coupled-inertia damper. There are two other types of dampers in use. The simplest damper provides merely a fixed viscous load. Such a damper has the same effect, from a linear point of view, as tachometer feedback, but it absorbs a great deal of the motor power. It is inferior in operation to the viscous-coupled-inertia damper, because it absorbs much more motor power, and provides

a much lower velocity constant. The other type of damper in use is similar to the viscous-coupled-inertia damper, but also has a spring coupled between the inertia slug and the motor shaft. This produces a quadratic lead-network effect in the loop transfer-function, which allows a lower inertia ratio for a given amount of phase lead. On the other hand, this damper has the disadvantage that it is much more sensitive to temperature changes than is the simple viscous-coupled-inertia damper.

Comparison of Systems

In comparing the systems, it is very important to consider whether the systems are being designed for the maximum possible crossover frequency, or whether the crossover frequency is to be maintained quite low in order for the system to perform filtering. Let us first consider the maximum bandwidth case.

Wide-Bandwidth Applications

The transfer functions that have been considered do not show any inherent limitation upon the crossover frequencies that are achievable in the systems. In practice, however, there are a number of sources of high-frequency phase lag that limit the crossover frequencies that can be achieved with adequate stability. Since the low-frequency double integrations in the loop transfer-functions of the damper and lead-network systems add phase lag at the crossover frequency, the tachometer system can achieve a slightly higher crossover frequency with a given amount of high-frequency phase lag.

Nevertheless, it is adequate for our analysis to assume that, for all three systems, the crossover frequencies are essentially the same and also that the upper break frequencies for the three transfer functions are set equal. These upper break frequencies, designated ω_u, are seen from Fig. 3 to be equal to

$$\text{Lead Net: } \omega_u = \alpha\omega_l \tag{36}$$

$$\text{Tach: } \omega_u = \omega_t + \omega_m = (f_t + f_m)/J_m \tag{37}$$

$$\text{Damper: } \omega_u = \alpha\omega_d. \tag{38}$$

The velocity constants for the three systems are summarized below:

$$\text{Lead Net: } K_v = (\omega_l/\omega_m)\omega_c \tag{17}$$

$$\text{Tach: } K_v = \omega_c \tag{27}$$

$$\text{Damper: } K_v = (\alpha\omega_d/\omega_m)\omega_c. \tag{33}$$

In practice ω_d and ω_l would be roughly the same. Thus, for a given ω_c, the velocity constant of the lead-network system is the ratio (ω_l/ω_m) greater than that of the tachometer system, and the velocity constant of the damper system is α times that of the lead-network system. In practice, the velocity constant of a damper system is often so high that constant-velocity components of error are negligible, and so the system may be considered to have an infinite velocity constant.

The torque constants can best be compared by expressing them in terms of the upper break frequency ω_u, given in (36) to (38). This gives

$$\text{Lead Net: } K_T = (1/\alpha)J_0\omega_u\omega_c \tag{39}$$

$$\left.\begin{array}{l}\text{Tach:}\\ \text{Damper:}\end{array}\right\} K_T = J_0\omega_u\omega_c \tag{40}$$

where J_0 is motor-plus-load inertia reflected to the output shaft and is equal to n^2J_m. If it is assumed that ω_u and ω_c are the same for the three systems, the tachometer and damper systems have the same torque constant (stiffness), but that of the lead-network system is smaller by the factor α.

Thus, for wide-bandwidth applications, the lead-network system has a comparatively high velocity-constant, but a low torque-constant; the tachometer system has a low velocity-constant, but a high torque-constant; and the damper system has both a very high velocity-constant and a high torque-constant.

It is often desirable to relate the torque-constant to the motor shaft rather than the output shaft. The motor-shaft torque-constant is defined as the ratio of a load torque applied to the motor shaft to the resultant angular displacement of that shaft. The expression for the motor-shaft torque-constant is obtained by replacing J_0 in (39) and (40) by J_m, the inertia reflected to the motor shaft. In fact the torque constant related to any point on the gear train is obtained by replacing J_0 by the inertia reflected to that point.

The low torque-constant of the lead-network system and the low velocity-constant of the tachometer system can be increased by adding low-frequency integral networks. However, for ac servo applications, this is inconvenient because it required demodulation and remodulation, which adds noise, drift, and phase lag to the system and may greatly increase its complexity (ac tuned networks are not practical for achieving an integral-network effect because of the extremely narrow network bandwidth required). In addition to the problem of implementation, a more basic difficulty of increasing the torque constant of the lead-network system by an integral network is that the action of that integral network can be effective only at quite low frequencies. Consequently, even though the integral network is effective in offsetting constant load torques, it is quite ineffective against intermittent torques, such as coulomb friction in the gear train. In the presence of coulomb friction, an integral network in a lead-network system often causes a low-frequency small-amplitude oscillation.

In the tachometer system, a simple way of increasing the velocity constant is to place a high-pass filter in the tachometer feedback path. This achieves an integral network effect by eliminating the damping action of the tachometer at low frequencies in the same manner as rotation of the floating inertia slug eliminates the damping action of the damper at low frequencies. Consequently, the transfer function of a tachometer system

having a high-pass tachometer filter is identical to that of the damper system in Fig. 3(c). The break frequency ω_d of the damper corresponds to the break frequency of the high-pass filter. The high-pass filter does not require that the amplifier gain be increased nor does it affect the torque constant.

It is also possible to increase the velocity constant of the tachometer system by adding a cascaded integral network, but this requires a proportionate increase in the amplifier gain and also increases the torque constant by the same amount. For a wide-bandwidth tachometer system, the torque constant is usually adequate without an additional integral network.

Narrow-Bandwidth Applications

Often a servomechanism is designed for narrow-bandwidth operation so that it can act as a filter upon noise in the reference input signal. For such applications, the tachometer system is superior because the inner tachometer bandwidth can be made quite wide, and the outer bandwidth, quite narrow. A wide inner bandwidth of the tachometer produces high stiffness even for oscillating and intermittent load torques. The low outer bandwidth provides the proper filtering. The torque-constant relation in (39) still applies; but ω_u (which is essentially equal to the crossover frequency ω_t of the tachometer loop) is set as high as possible, while ω_c, the crossover frequency of the position loop, is set quite low in order to achieve filtering.

In the damper system, ω_c should never be very much less than ω_u because such a setting would result in an excessive amount of power loss by the damper. In the lead-network system, ω_c can be set much less than ω_u, but this merely has the effect of increasing the high-frequency noise transmission. In (39), the torque constant stays the same with an increase in ω_u because the attenuation ratio α increases by the same amount as ω_u. A better way to see this is to examine the expression for K_T that was given in (20). It shows that for a given ω_c, the torque constant can be increased only by increasing ω_l, which would decrease the stability of the system.

Saturation

A fundamental fault of the damper is that under transient conditions, it draws power from the motor. This fault has two bad effects. First, it decreases the acceleration capabilities of the motor by the factor α, and second, it produces a very long nonlinear oscillating transient following a large saturating step input.

The decrease in acceleration capabilities can be analyzed as follows. The reciprocal of (11) relates the required motor torque to a given output velocity. If it is assumed for simplicity that ω_m is zero, the reciprocal of (11) becomes

$$\frac{T_m}{\Omega_m} = \frac{sJ_m(s+\alpha\omega_d)}{(s+\omega_d)}. \tag{41}$$

Designate as A_m the motor acceleration, which is equal to $s\Omega_m$. Eq. (41) then becomes

$$\frac{T_m}{A_m} = (\alpha J_m)\frac{(s/\alpha\omega_d)+1}{(s/\omega_d)+1}. \tag{42}$$

A plot of this function is shown in Fig. 4(a). For frequencies below ω_d, the ratio T_m/A_m is αJ_m. Therefore, if the motor is to maintain a given acceleration longer than the period $1/\omega_d$, it must supply enough torque to accelerate the mass αJ_m which is the total mass coupled to the motor shaft. For acceleration components maintained for shorter periods of time, however, the torque requirements are less. For acceleration components maintained for periods less than $1/\alpha\omega_d$, the effective inertia load is only the direct-coupled motor inertia J_m.

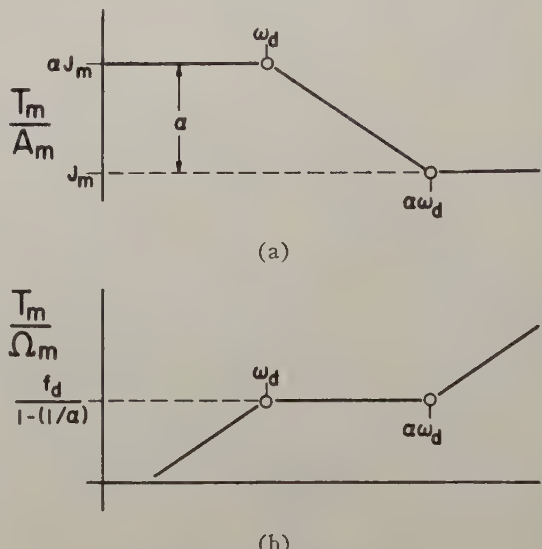


Fig. 4—Torque requirements.

For frequencies greater than ω_d, but less than $\alpha\omega_d$, the damper acts like a viscous load with a damping ratio of roughly f_d. To show this, express (41) as follows:

$$\frac{T_m}{\Omega_m} = f_d\left(\frac{\alpha}{\alpha-1}\right)\frac{1+(s/\alpha\omega_d)}{1+(\omega_d/s)}. \tag{43}$$

A plot of this function is shown in Fig. 4(b). A time-domain interpretation of (43) is that for an acceleration component maintained for a shorter period than $1/\omega_d$, but longer than $1/\alpha\omega_d$, the required motor torque may be calculated by integrating that acceleration component (to obtain the velocity) and multiplying the velocity by the damping factor $f_d(\alpha-1)/\alpha$, or roughly by f_d.

A serious drawback of the damper system is its poor transient response following a very large step input. If the input is large enough to maintain the motor saturated for a significant period of time, the damper is able to pick up a great deal of angular momentum, enough to drag the motor past the point of zero error, deep into

saturation in the opposite direction. As the motor reverses speed, the slug again picks up velocity in the return direction, and repeats the procedure. This may repeat for a number of cycles before the system comes to rest.

A good way to eliminate this poor transient response for large steps is to build a clutching mechanism that declutches the damper from the motor during saturated conditions. A description of the design and operation of such a clutching mechanism is given [1].

Noise

For all three systems, the amplifier gain K_a is given by

$$K_a = (nJ_m/K_m)\omega_u\omega_c. \tag{44}$$

The amplifier gain determines the amount of high-frequency noise transmitted to the motor. As shown in Fig. 2, the ratio V_m/θ_e at high frequencies is K_a for all three systems. Thus, for equal values of ω_c, and ω_u, all three systems require equal amplifier gains and are equally noisy, as would be expected.

Lead-network systems are often considered to be much more noisy than damper and tachometer systems. This is primarily because a major requirement in servo design is adequate stiffness (torque constant). For the lead-network system, the amplifier gain K_a is related to the torque constant K_T by

$$K_a = \alpha K_T/nK_m \text{ (lead net).} \tag{45}$$

But, for the tachometer and damper systems, the relation is

$$K_a = K_T/nK_m \text{ (damper and tachometer).} \tag{46}$$

Thus, for a given torque constant, the lead network requires α times the amplifier gain K_a, and hence has α times as much noise on the motor.

Gear backlash is the most troublesome in lead-network systems. That is because the major compensation path is through the gear train in lead-network systems, whereas damper and tachometer systems have a much tighter compensation path around the motor. Thus, a lead-network system tends to chatter more because of backlash. The system with the least tendency to chatter is the damper system, because the direct-coupled viscous load tends to damp out any mechanical gear-train vibrations.

Transient Behavior

The parameters that have been used to define the dynamic behavior of the systems are the gain-crossover frequency ω_{cg} (or the asymptote-crossover frequency ω_c) and the velocity constant K_v. The significance of these parameters in terms of transient behavior is shown by the approximate transient responses in Fig. 5. System 1 could represent the tachometer system and System 2 either the lead-network or damper system.

In Fig. 5(b), the step responses both rise to 63 per cent of the final values in the time $1/\omega_{cg}$. In Fig. 5(c), the maximum error for a unit-ramp input is the same for both, and also equal to $1/\omega_{cg}$. These two approximate relationships hold reasonably well for all practical feedback-control loops, which is the reason why gain crossover is such a good criterion for bandwidth.

The final, or steady-state, values of the unit-ramp error responses in Fig. 5(c) are equal to the reciprocal of the corresponding velocity constants. Fig. 5(a) shows that the velocity constant for System 1 is ω_c and for System 2 is $\beta\omega_c$. Consequently, the steady-state error for System 2 in Fig. 5(c) is $1/\beta$ times that for System 1, although both systems have the same maximum error. The reduction in the steady-state error of System 2 is due to the action of its low-frequency double integration. It is logical that the double integration in System 2 cannot appreciably affect the maximum value of the error for a ramp input, because the double integration has effect only at low frequencies.

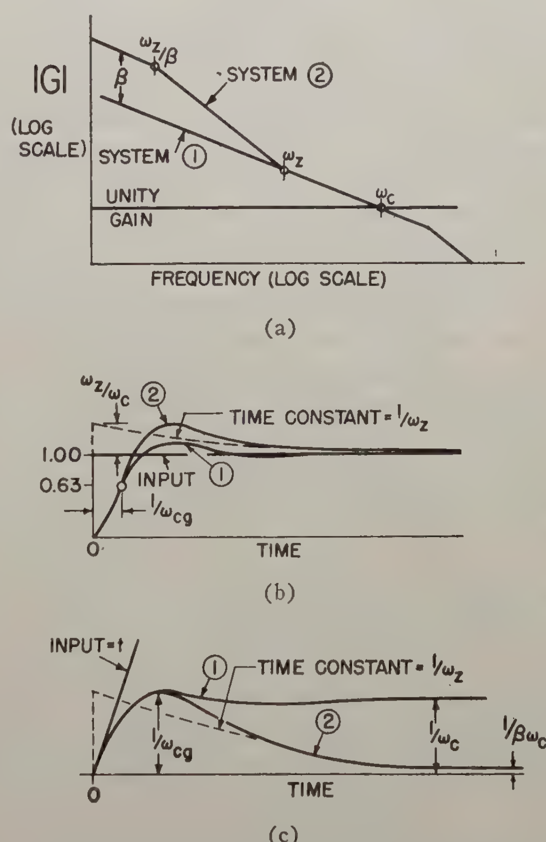


Fig. 5—Approximate transient responses. (a) Frequency-response asymptotes. (b) Unit step response. (c) Unit ramp error-response.

The tail in the ramp-error response of System 2 essentially follows an exponential with the time constant $1/\omega_z$, where ω_z represents the upper break frequency of the double integration, and corresponds to an open-loop zero of G. Thus, the higher the frequency ω_z, the more rapid is the reduction of the velocity error from its maximum value—in other words, the more rapid is the action of the double integration.

The double integration of System 2 improves the

ramp response by reducing the steady-state error, but it harms the step response by increasing the overshoot and adding a tail. Fig. 5(b) shows that the double integration adds to the step response an exponential, having a time constant of roughly $1/\omega_z$ and a magnitude of roughly ω_z/ω_c.

The reason for the double integration producing an exponential with a time constant roughly equal to $1/\omega_z$ can be found by considering the closed-loop poles of the system. It is shown that low-frequency zeros in the loop transfer-function, G, result in closed-loop poles, which are shifted somewhat from these zeros [2]. Therefore, System 2 must have a closed-loop pole roughly equal to $-\omega_z$, and this pole must add an exponential to each transient response with a time constant roughly equal to $1/\omega_z$.

The relations described above relating transient and frequency response are also considered in more detail [3].

Conclusions

The paper has tried to provide a general basis for comparing compensation techniques used in electric positional servomechanism. The comparison has not considered integral compensation as a separate compensation technique, because integral compensation is generally used in such servomechanisms merely to improve low-frequency performance (increase stiffness and reduce low-frequency dynamic errors), and as such may be considered as merely a supplement to the basic compensation techniques that have been discussed. Since there are many different configurations that an electric servomechanism can take, and many practical effects that have not been considered in the paper, it is very difficult to state general conclusions that are without exception. Nevertheless, the following is an attempt to summarize the comparison of the three compensation techniques.

Lead-network compensation is simple and inexpensive, and allows the servo to achieve a wide bandwidth, a relatively high velocity-constant, and reasonable torque-stiffness. However, the stiffness is still considerably lower than can be achieved with damper or tachometer compensation. Consequently, lead-network compensation is often not adequate for high-accuracy applications, because it results in excessive errors due to load torques and static friction.

Tachometer compensation can provide wide bandwidth and high stiffness. When a high-pass filter is placed in the tachometer feedback path, it also can provide a very high velocity constant. On the other hand, to achieve this high performance requires a rather expensive tachometer with a low noise level and low quadrature.

The viscous-coupled inertia damper provides a wide bandwidth, and very high stiffness and velocity constant, with rather simple electronic circuitry. Its main disadvantages are that it can tolerate only a small reflected inertia from the load, and it greatly restricts the power capability of the motor under transient conditions. Consequently, inertial-damper compensation is generally applicable only to instrument servomechanisms, but in such applications it can perform exceedingly well. It can achieve significantly better performance than tachometer feedback, because tachometer compensation is limited by noise and dynamic lags in the tachometer loop. Damper compensation allows wider bandwidth and a correspondingly higher torque stiffness. The author has experienced a factor of 3 difference in the torque stiffness between tachometer and damper compensation for equivalent instrument servomechanisms.

The viscous damping in an inertial damper can be provided either by fluid coupling or by magnetic coupling. A practical difficulty with using viscous fluid is that the fluid requires some sort of temperature control if the servomechanism is subject to wide extremes of temperature. The magnetic-coupled damper has no temperature problems, but is severely limited in the viscosity it can provide for a given value of slug inertia, because the slug is a permanent magnet which provides all the magnetic flux. Damping is produced by the magnet coupling between the slug and an eddy current disk on the motor shaft. The limitation in the viscosity-to-inertia ratio severely restricts the value of damper break-frequency ω_d. Because of this, the resultant bandwidth and torque constant are considerably lower than can be achieved by a viscous-coupled inertial damper for most 400-cps instrument-servo applications.

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The Analysis of Sampled-Data Control Systems with a Periodically Time-Varying Sampling Rate*

E. I. JURY† AND F. J. MULLIN‡

Summary—The z-transform is used to solve sampled-data systems which have a periodically time-varying sampling rate, *i.e.*, systems which have a repetitive sampling pattern in which the time duration between the individual samples is not constant. Such systems are described by linear difference equations with periodic coefficients; however, the difference equation which describes the system at sampling instants corresponding to KN, where N is the period of the coefficients of the difference equation, and $K=0, 1, 2, \cdots$, is a linear difference equation with constant coefficients. Thus by forming a series of difference equations which individually describe the system at sampling instants corresponding to KN, $KN+1$, $KN+2, \cdots$, $(K+1)N-1$, the time varying features of the system are in essence removed from the analysis and the z-transform can be used to solve the resulting constant coefficient difference equations. Also, the response between the sampling instants can be found using the solutions of these difference equations.

The method presented is straightforward and can be used to analyze any linear sampled-data system with a periodic sampling pattern. Such a condition could occur, for example, when a computer is time shared by more than one system or in some telemetering devices which periodically give to control systems information on quantities being monitored but in which the desired information is not available at equally spaced intervals of time. This method can also be used to obtain an approximate solution for the output of any linear system which is excited by a periodic but nonsinusoidal forcing function and, because of the flexibility of the sampling pattern, should give more accurate results than an approximation which uses equally spaced samples.

In this analysis, only periodicity of the sampling pattern is assumed, and no relationship between the individual sampling intervals is required. A few examples have been introduced to illustrate the analytical procedure and the features of the response of a system to sinusoidal inputs is indicated in one of the examples.

Introduction

LINEAR sampled-data systems have been extensively studied during the past several years. They have been analyzed using the impulsive response method,[1] the z-transform and the modified z-transform[2–4] and more recently with state vectors and matrices.[5,6] In fact, the analysis of sampled-data systems is understood well enough to permit a great deal of work to be done on the synthesis and optimization of these systems, and Freeman and Lowenchuss[7] offer a complete bibliography covering both the analysis and synthesis of such systems.

Primarily, all of the systems which have been investigated up to the present time were those with a constant sampling frequency. Farmanfarma[8] and Kranc[9] have studied systems in which two or more samplers operate at constant frequencies and quite recently, Hufnagel[10] introduced a procedure for the analysis of cyclic-rate (periodically time varying) sampled-data feedback systems. His method is based on the modified z-transform where the techniques developed for constant sampling rate systems are readily applicable.

In this paper, we will discuss the same systems as Hufnagel but use difference equations, rather than the modified z-transform, as the starting point. We shall also use the z-transform although matrix methods could be just as easily used. This choice is simply a matter of preference.

Mathematical Background

Consider the sampled-data system shown in Fig. 1 in which T_n is the time in seconds between the nth and $(n+1)$th sampling instants. We assume that T_n is periodic; that is, $T_n = T_{n+v}$ where v is the number of sampling intervals required for the pattern to repeat itself. The period of the sampling pattern is T. For example, if $v=4$, then for the time function, say $e(t)$, shown in Fig. 2(a), the corresponding sampled time function, $e^*(t)$, is shown in Fig. 2(b). The periodicity of the sampling pattern shown in Fig. 2 should be observed since it is this feature which allows the output of the system to be expressed in a closed form.

These systems can be analyzed as follows. Let E_n denote the output of the hold circuit shown in Fig. 1 for $t_n \leq t \leq t_{n+1}$ and $c(t)$, the continuous output of the system. Then, from the differential equation (or transfer function) relating $c(t)$ to E_n, the continuous output can be written, say for the $(n+1)$th sampling interval, as:

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[9] G. M. Kranc, "Input-output analysis of multirate feedback systems," IRE TRANS. ON AUTOMATIC CONTROL, vol. PGAC-3, pp. 21–28; November, 1957.

[10] R. E. Hufnagel, "Analysis of cyclic rate sampled-data feedback control systems," *Trans. AIEE*, vol. 77, pp. 421–423; November, 1958.

$$\underset{t_n \le t \le t_{n+1}}{c(t)} = E_n g(t - t_n) + \sum_{p=0}^{q-1} f_p(t - t_n) \left(\frac{d^p c(t)}{dt^p} \bigg|_{t=t_n} \right). \quad (1)$$

In the above equation, t_n is the actual time at the start of the $(n+1)$th sampling interval (the first sampling interval starts at $t=0$), $g(t)$ is the step function response of $G(s)$ shown in Fig. 1, q is the order of the system, and $f_p(t)$ are time functions which result from initial conditions and the solution of the differential equation relating $c(t)$ to E_n with E_n set equal to zero. In other words, the $f_p(t)$ and the various derivative terms appear in the solution because of the initial conditions on $c(t)$ and its $(q-1]$ derivatives which must be satisfied at the start of the $(n+1)$th sampling interval. [See, for example, (5) and (21).] Since the system is of order q, we must obtain q first order equations to completely describe the system, and they may be obtained by differentiating the above equation $(q-1)$ times. They are of the form

$$\underset{t_n \le t \le t_{n+1}}{\frac{dc}{dt}} \triangleq c^{(1)}(t) = E_n g^{(1)}(t - t_n) + \sum_{p=0}^{q-1} f_p^{(1)}(t - t_n) \left(\frac{d^p c(t)}{dt^p} \bigg|_{t=t_n} \right) \quad (2a)$$

$$\underset{t_n \le t \le t_{n+1}}{\frac{d^2c}{dt^2}} \triangleq c^{(2)}(t) = E_n g^{(2)}(t - t_n) + \sum_{p=0}^{q-1} f_p^{(2)}(t - t_n) \left(\frac{d^p c(t)}{dt^p} \bigg|_{t=t_n} \right) \quad (2b)$$

. . . .

$$\frac{d^{q-1}c}{dt^{q-1}} \triangleq c^{(q-1)}(t) = E_n g^{(q-1)}(t - t_n) + \sum_{p=0}^{q-1} f_p^{(q-1)}(t - t_n) \left(\frac{d^p c(t)}{dt^p} \bigg|_{t=t_n} \right) \quad (2(q-1))$$

In these equations we have used the following notations:

$$\frac{d^k g(t)}{dt^k} \triangleq g^{(k)}(t); \quad \frac{d^k f_p(t)}{dt^k} \triangleq f^{(k)}(t) \quad k = 0, 1, 2, \cdots (q-1).$$

For notational convenience let us also define:

$$\frac{d^k c(t)}{dt^k}\bigg|_{t=t_n} \triangleq C_n^{(k)}, \qquad k = 0, 1, 2, \cdots (q-1).$$

If we let $t=t_{n+1}$ in (1) to (2(q−1)) and note that $t_{n+1}-t_n=T_n$, we obtain q difference equations which characterize the system at the sampling instants. For a unity feedback system these equations are:

$$C_{n+1} = (R_n - C_n)g(T_n) + \sum_{p=0}^{q-1} f_p(T_n) C_n^{(p)} \quad (3a)$$

$$C_{n+1}^{(1)} = (R_n - C_n)g^{(1)}(T_n) + \sum_{p=0}^{q-1} f_p^{(1)}(T_n) C_n^{(p)} \quad (3b)$$

. . . .

$$C_{n+1}^{(q-1)} = (R_n - C_n)g^{(q-1)}(T_n) + \sum_{p=0}^{q-1} f_p^{(q-1)}(T_n) C_n^{(p)} \quad (3q)$$

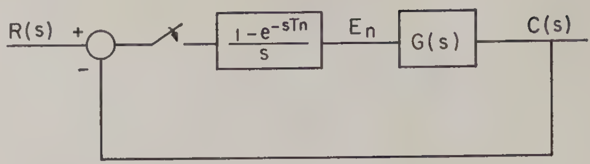

Fig. 1—A sampled-data system which has a time varying sampling rate.

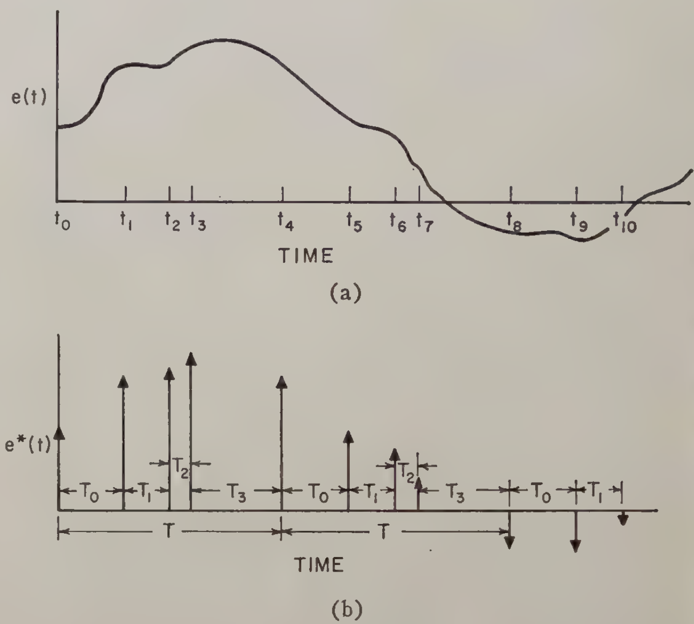

Fig. 2—(a) A continuous time function with the sampling times marked on the time axis. (b) The sampled time function when the sampler has a periodically time varying sampling rate.

We have described the system over an arbitrary interval in (3a–q). We also know that $T_n = T_{n+v}$ because of the periodicity of the sampling pattern. Therefore, since T_n is periodic, (3a–q) are q linear difference equations with periodic coefficients where the periodicity of the coefficients is v sampling intervals or T seconds on the real time axis. And to obtain a solution, we must solve q simultaneous linear difference equations with periodic coefficients.

This can be done in a straightforward manner using either the z-transform[11] or matrix methods;[12] the reader is referred to the Appendix for the z-transform method of solution. It should be pointed out here, however, that although this later method is straightforward, the amount of labor required to obtain a solution increases as the number of samples in the sampling pattern becomes larger in number.

Example 1

In Fig. 1 let $G(s) = 1.0/s(s+2)$. Assume that the sampling pattern repeats itself after 3.0 seconds ($T=3.0$

[11] F. J. Mullin and E. I. Jury, "A note on the operational solution of linear difference equations," *J. Franklin Inst.*, vol. 266, pp. 189–205; September, 1958.

[12] B. Friedland, "Theory of time-varying sampled-data systems," Columbia University, Electronics Res. Lab., New York, N. Y., Tech. Rep. No. T-19/B; April, 1957.

seconds) and that the individual sampling durations are given by:

$$T_n = \begin{cases} 0.5 \text{ sec.} & n = 0, 4, 8, 12, \cdots \\ 0.5 \text{ sec.} & n = 1, 5, 9, 13, \cdots \\ 1.0 \text{ sec.} & n = 2, 6, 10, 15, \cdots \\ 1.0 \text{ sec.} & n = 3, 7, 11, 15, \cdots . \end{cases}$$

$(v=4)$

A graphical interpretation of this sampling pattern is shown in Fig. 3. It is desired to find the continuous response of the system.

For this choice of $G(s)$ the differential equation relating $c(t)$ to E_n is:

$$\frac{d^2c}{dt^2} + 2\frac{dc}{dt} = E_n \qquad t_n \le t \le t_{n+1}. \tag{4}$$

The solution to (4) may be written:

$$\underset{t_n \le t \le t_{n+1}}{c(t)} = 0.25E_n[2(t - t_n) + e^{-2(t-t_n)} - 1] + C_n + 0.5C_n^{(1)}(1 - e^{-2(t-t_n)}). \tag{5}$$

Differentiating (5) gives:

$$c^{(1)}(t) = 0.5E_n(1 - e^{-2(t-t_n)}) + C_n^{(1)}(e^{-2(t-t_n)}). \tag{6}$$

Setting $t = t_{n+1}$ in (5) and (6) gives the difference equations which describe the system; with $E_n = R_n - C_n$, the equations become:

$$\begin{aligned} C_{n+1} &= 0.25R_n(2T_n + e^{-2T_n} - 1) \\ &\quad + 0.25C_n(5 - e^{-2T_n} - 2T_n) \\ &\quad + 0.5C_n^{(1)}(1 - e^{-2T_n}) \end{aligned} \tag{7}$$

$$\begin{aligned} C_{n+1}^{(1)} &= 0.5R_n(1 - e^{-2T_n}) - 0.5C_n(1 - e^{-2T_n}) \\ &\quad + C_n^{(1)}(e^{-2T_n}). \end{aligned} \tag{8}$$

Since $T_n = T_{n+4}$ for the example above, (7) and (8) are linear difference equations with periodic coefficients, and even though there are only two values of T_n, the period of repetition of the over-all sampling pattern is four sampling intervals, and hence the coefficients of R_n, C_n and $C_n^{(1)}$ in (7) and (8) repeat themselves after four sampling intervals. For the values of T_n chosen in this example, (7) and (8) become:

$T_n = 0.5$ second

$$C_{n+1} = 0.092R_n + 0.908C_n + 0.316C_n^{(1)} \tag{9a}$$

$$C_{n+1}^{(1)} = 0.316R_n - 0.316C_n + 0.368C_n^{(1)} \tag{9b}$$

$T_n = 1.0$ second

$$C_{n+1} = 0.284R_n + 0.716C_n + 0.432C_n^{(1)} \tag{10a}$$

$$C_{n+1}^{(1)} = 0.432R_n - 0.432C_n + 0.135C_n^{(1)}. \tag{10b}$$

At this point, it becomes convenient to make the following notational definitions. Since we have a repetition period of four samples we define:

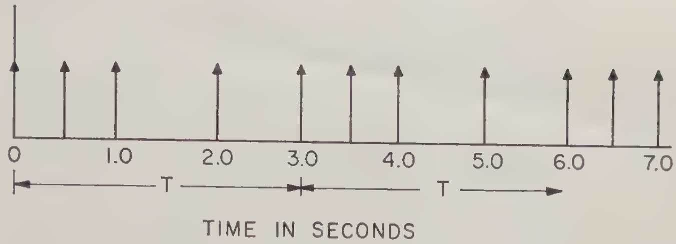

Fig. 3—The sampling pattern used in Example 1.

$$C_n = \begin{cases} C_{4K} & n = 0, 4, 8, 12, \cdots \\ C_{4K+1} & n = 1, 5, 9, 13, \cdots \\ C_{4K+2} & n = 2, 6, 10, 14, \cdots \\ C_{4K+3} & n = 3, 7, 11, 15, \cdots \end{cases}$$

where $K = 0, 1, 2, 3, \cdots$. Having done this, we can follow the operational procedure which is briefly outlined in the Appendix to obtain the solution of (9) and (10). Then, we would find:

$$C_{4(K+1)}(z) = \frac{z^4C_0 + R_a(z) + 0.145C_{4K}^{(1)}(z)}{z^4 - 0.0878} \tag{11}$$

$$C_{4(K+1)}^{(1)}(z) = \frac{z^4C_0^{(1)} + R_b(z) - 0.199C_{4K}(z)}{z^4 + 0.154} \tag{12}$$

in which

$$\begin{aligned} R_a(z) &= 0.284R_{4K+3}(z) + 0.390R_{4K+2}(z) + 0.146R_{4K+1}(z) \\ &\quad + 0.0919R_{4K}(z) \\ R_b(z) &= 0.432R_{4K+3}(z) - 0.0642R_{4K+2}(z) - 0.0871R_{4K+1}(z) \\ &\quad - 0.0821R_{4K}(z). \end{aligned}$$

With the initial conditions $C_0 = C_0^{(1)} = 0$, the simultaneous solution of (11) and (12) gives:

$$C_{4K}(z) = \frac{R_a(z)(z^4 + 0.154) + 0.145R_b(z)}{z^8 + 0.0667z^4 + 0.152} \tag{13}$$

$$C_{4K}^{(1)}(z) = \frac{R_b(z)(z^4 - 0.0878) - 0.199R_a(z)}{z^8 + 0.0667z^4 + 0.152}. \tag{14}$$

The inverse z-transforms of these two equations, assuming the input is a unit step,[8] are:

$$C_{4K} = 1.0 - 1.435(0.123)^K \cos(105.7K - 45.6°) \tag{15a}$$

$$C_{4K}^{(1)} = 1.677(0.123)^K \cos(105.7K - 90°). \tag{16a}$$

These two equations give the sampled response for $n = 0, 4, 8$, etc. or for $t = 0$, 3 seconds, 6 seconds, etc. To find the response for $n = 1, 5, 9$, etc. or $t = 0.5$ second, 3.5 seconds, 6.5 seconds, etc., we can use (9a) and (9b) with (15a) and (16a) and find the expressions for C_{4K+1} and $C_{4K+1}^{(1)}$ in terms of C_{4K} and $C_{4K}^{(1)}$. With C_{4K+1} and $C_{4K+1}^{(1)}$ determined, C_{4K+2} and $C_{4K+2}^{(1)}$ can be found in the same fashion. When this procedure is carried out, we find:

$$C_{4K+1} = 1.0 - 0.993(0.123)^K \cos(105.7K - 23.8°) \quad (15b)$$

$$C_{4K+2} = 1.0 - 0.725(0.123)^K \cos(105.7K - 5.27°) \quad (15c)$$

$$C_{4K+3} = 1.0 - 0.379(0.123)^K \cos(105.7K - 24.9°) \quad (15d)$$

$$C_{4K+1}^{(1)} = 0.994(0.123)^K \cos(105.7K - 71.4°) \quad (16b)$$

$$C_{4K+2}^{(1)} = 0.621(0.123)^K \cos(105.7K - 49.5°) \quad (16c)$$

$$C_{4K+3}^{(1)} = 0.378(0.123)^K \cos(105.7K - 14.2°). \quad (16d)$$

Eqs. (15a–d) give the sampled response of this system. The complete response can be determined quite simply if we divide the time axis into four periodic intervals as follows:

$$\begin{aligned} K &\le t \le K + 0.5 \\ K + 0.5 &\le t \le K + 1.0 \\ K + 1.0 &\le t \le K + 2.0 \\ K + 2.0 &\le t \le K + 3.0 \qquad K = 0, 1, 2, \cdots. \end{aligned}$$

This division is motivated by a consideration of the duration of the individual sampling intervals which occur during one period of the sampling pattern. To find the response over the first interval, we simply substitute (15a) and (16a) for C_n and $C_n^{(1)}$ in (5), replace $(t-t_n)$ by $mT_0=0.5\ m$, $0 \le m \le 1$, and obtain the continuous time response of the system over the first interval. This essentially results in the modified z-transform[4] of the output. To show some detail, we re-write (5) as follows:

$$\begin{aligned} \underset{t_n \le t \le t_{n+1}}{c(t)} &= 0.25R_n(2(t - t_n) + e^{-2(t-t_n)} - 1) \\ &\quad + 0.25C_n(5 - e^{-2(t-t_n)} - 2(t - t_n)) \\ &\quad + 0.5C_n^{(1)}(1 - e^{-2(t-t_n)}). \end{aligned} \quad (5')$$

Making the substitutions mentioned previously gives:

$$\begin{aligned} \underset{\substack{K \le t \le K+0.5m \\ 0 \le m \le 1}}{c(t)} &= 0.5(m + e^{-m} - 1) \\ &\quad + 0.25(5 - e^{-m} - m)[1.0 - 1.435(0.123)^K \\ &\qquad \times \cos(105.7K - 45.6°)] \\ &\quad + 0.50(1 - e^{-m})[1.677(0.123)^K \cos(105.7K - 90°)]. \end{aligned}$$

Simplifying this expression a little shows:

$$\begin{aligned} \underset{\substack{K \le t \le K+0.5m \\ 0 \le m \le 1}}{c(t)} &= 1.0-(5-e^{-m}-m)[0.359(0.123)^K \cos(105.7K-45.6°)] \\ &\quad +(1-e^{-m})[0.839(0.123)^K \cos(105.7K-90°)]. \end{aligned} \quad (17a)$$

For the second interval, C_{4K+1} and $C_{4K+1}^{(1)}$ are substituted into (5) and $(t-t_n)$ is replaced by $mT_1=0.5m$; for the third interval, C_{4K+2} and $C_{4K+1}^{(1)}$ are used and $(t-t_n)$ is replaced by $mT_2=m$, and so forth. The final results can be written:

$$\begin{aligned} \underset{\substack{K+0.5 \le t \le K+0.5(1+m) \\ 0 \le m \le 1}}{c(t)} &= 1.0-(5-e^{-m}-m)[0.248(0.123)^K \cos(105.7K-23.8°)] \\ &\quad +(1-e^{-m})[0.497(0.123)^K \cos(105.7K-71.4°)] \end{aligned} \quad (17b)$$

$$\begin{aligned} \underset{\substack{K+1 \le t \le K+1+m \\ 0 \le m \le 1}}{c(t)} &= 1.0-(5-e^{-m}-m)[0.811(0.123)^K \cos(105.7K-5.27°)] \\ &\quad +(1-e^{-m})[0.311(0.123)^K \cos(105.7K-49.5°)] \end{aligned} \quad (17c)$$

$$\begin{aligned} \underset{\substack{K+2 \le t \le K+2+m \\ 0 \le m \le 1}}{c(t)} &= 1.0-(5-e^{-m}-m)[0.0948(0.123)^K \cos(105.7K-24.9°)] \\ &\quad +(1-e^{-m})[0.189(0.123)^K \cos(105.7K-14.2°)]. \end{aligned} \quad (17d)$$

The continuous response is shown in Fig. 4.

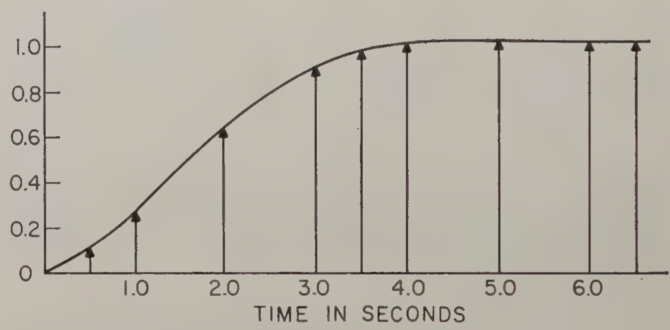

Fig. 4—The continuous output of Example 1.

Example 2

If a sine wave is applied to the input of the system shown in Fig. 3, the response of the system is quite different from that of a sampled-data system with a constant sampling rate. For example, let $G(s)$ and T_n be the same as in Example 1, and let the input to the system, $r(t)=1.0 \sin \pi/2t$. Then the transforms of the sampled output of the system and its first derivative are again given by (13) and (14), but it should be noted that $R_a(z)$ and $R_b(z)$ are not the same as they were in Example 1; actually for this example:

$$R_{4K}(z) = \frac{z^4}{z^8+1} \qquad R_{4K+2} = \frac{z^8}{z^8+1}$$

$$R_{4K+1}(z) = \frac{0.707z^4(z^4-1)}{z^8+1} \qquad R_{4K+3}(z) = \frac{z^4}{z^8+1}.$$

If the same procedure that was used in the previous example is followed, we would find that when $r(t)=1.0 \sin \pi/2t$ and $C_0=C_0^{(1)}=0$:

$$C_{4K} = 0.428 \cos (90K - 106.5°) + 0.743(0.123)^K \cos [105.7K - 80.9°] \quad (18a)$$

$$C_{4K+1} = 0.377 \cos (90K - 134.8°) + 0.516(0.123)^K \cos [105.7K - 58.9°] \quad (18b)$$

$$C_{4K+2} = 0.244 \cos (90K - 171.8°) + 0.378(0.123)^K \cos [105.7K - 40.1°] \quad (18c)$$

$$C_{4K+3} = 0.264 \cos (90K + 43.9°) + 0.217(0.123)^K \cos [105.7K - 13.1°]. \quad (18d)$$

Also:

$$C_{4K}^{(1)} = 0.499 \cos (90K - 182.8°) + 0.868(0.123)^K \cos [105.7K + 55°] \quad (19a)$$

$$C_{4K+1}^{(1)} = 0.483 \cos (90K - 107°) + 0.516(0.123)^K \cos [105.7K + 73.5°] \quad (19b)$$

$$C_{4K+2}^{(1)} = 0.544 \cos (90K + 61.8°) + 0.323(0.123)^K \cos [105.7K + 95.2°] \quad (19c)$$

$$C_{4K+1}^{(1)} = 0.566 \cos (90K + 8.0°) + 0.186(0.123)^K \cos [105.7K + 130.6°]. \quad (19d)$$

The sampled output and the sampled derivative of the output are shown in Fig. 5(a) and Fig. 5(b); Fig. 5(c) shows the input sine wave so that an idea of the difference in the periods of the output and the input can be appreciated. It should be noted that the period of the input sine wave is 4 seconds, the period of the sampling pattern is 3 seconds, and the period of the response is 12 seconds. Although it was not investigated in this study, it is suspected that in general if the period of the input signal is α seconds and that of the sampling pattern, β seconds, then the period of the output will be $(\alpha\beta)$ seconds.

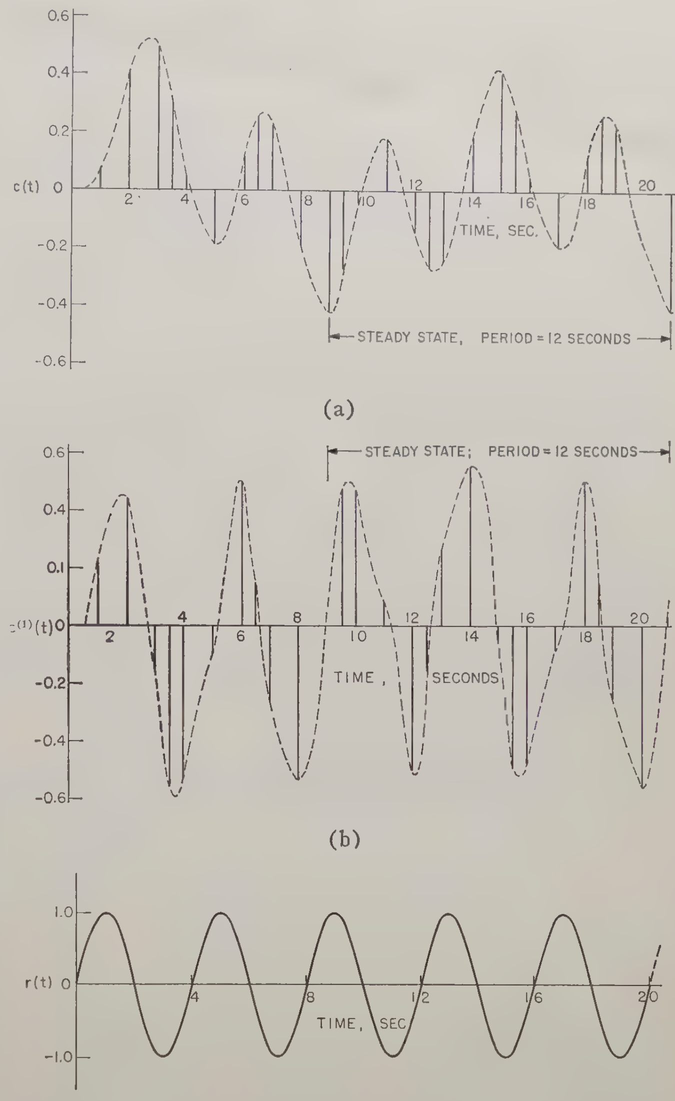

Fig. 5—(a) The sampled output of Example 2. (b) The sampled derivative of the output of Example 2. (c) The input, $r(t) = \sin \pi/2t$, used in Example 2.

Example 3

In Fig. 1 let:

$$G(s) = \frac{0.2}{s(s^2 + 2s + 2)} \qquad T_n = \begin{cases} 0.51 \text{ seconds } n \text{ even} \\ 0.33 \text{ seconds } n \text{ odd.} \end{cases}$$

$$T = 0.84 \text{ seconds.}$$

These values of T_n are chosen to indicate that there need not be any integer relationship between the various values of T_n: the only requirement is that the sampling rate be periodic.

For this example, the differential equation relating $c(t)$ to E_n is:

$$\frac{d^3c}{dt^2} + 2\frac{d^2c}{dt^2} + 2\frac{dc}{dt} = 0.2E_n \qquad t_n \le t \le t_{n+1}. \quad (20)$$

The response of the system during any sampling interval is:

$$\underset{\substack{t_n \le t \le t_{n+1} \\ t' = t - t_n}}{c(t)} = 0.1E_n(t' - 1 + e^{-t'} \cos t') + C_n + C_n^{(1)}(1 - e^{-t'} \cos t') + C_n^{(2)}[0.5 + 0.707e^{-t'} \sin (t' - 135°)] \quad (21)$$

$$\underset{\substack{t_n \le t \le t_{n+1} \\ t' = t - t_n}}{c^{(1)}(t)} = 0.1E_n[1 - \sqrt{2}e^{-t'} \sin (t' + 45°)] + C_n^{(1)}[\sqrt{2}e^{-t'} \sin (t' + 45°)] + C_n^{(2)}(e^{-t'} \sin t') \quad (22)$$

$$\underset{\substack{t_n \le t \le t_{n+1} \\ t' = t - t_n}}{c^{(2)}(t)} = 0.1E_n(2e^{-t'} \sin t') - C_n^{(1)}(2e^{-t'} \sin t') + C_n^{(2)}[\sqrt{2}e^{-t'} \cos (t' + 45°)]. \quad (23)$$

Setting $t=t_{n+1}$ in these equations gives three difference equations with coefficients of period two, and for the selected values of T_n these equations are:

$T=0.51$ second

$$C_{n+1}=0.003R_n+0.997C_n+0.476C_n^{(1)}+0.091C_n^{(2)} \quad \text{(24a)}$$

$$C_{n+1}^{(1)}=0.018R_n-0.018C_n+0.817C_n^{(1)}+0.293C_n^{(2)} \quad \text{(24b)}$$

$$C_{n+1}^{(2)}=0.057R_n-0.057C_n-0.586C_n^{(1)}+0.231C_n^{(2)}. \quad \text{(24c)}$$

$T=0.33$ second

$$C_{n+1}=0.001R_n-0.999C_n+0.320C_n^{(1)}+0.043C_n^{(2)} \quad \text{(25a)}$$

$$C_{n+1}^{(1)}=0.009R_n-0.009C_n+0.913C_n^{(1)}+0.233C_n^{(2)} \quad \text{(25b)}$$

$$C_{n+1}^{(2)}=0.047R_n-0.047C_n-0.466C_n^{(1)}+0.447C_n^{(2)}. \quad \text{(25c)}$$

These equations can be solved simultaneously as in the first example by letting C_{2K} represent the output at $t=0.84K$ second and C_{2K+1} represent the output at $t=0.84K+0.51$ second. With zero initial conditions the solutions for C_{2K} and C_{2K+1} are found to be

$$C_{2K} = 1.0 - 1.121(0.913)^K + 0.1221(0.462)^K \cos(45.9^K - 6.1°) \quad (26)$$

$$C_{2K+1} = 1.0 - 1.078(0.913)^K + 0.1094(0.462)^K \cos(49.8K - 4.2°). \quad (27)$$

The sampled response is shown in Fig. 6.

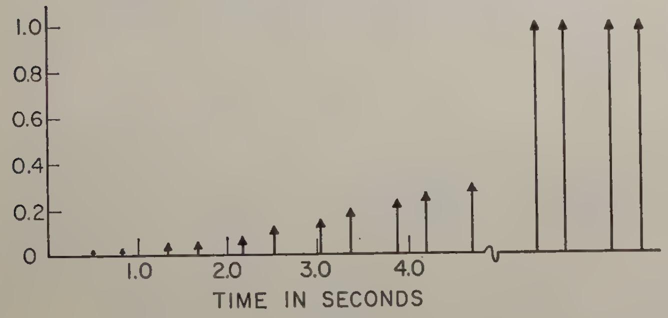

Fig. 6—The sampled output of Example 3.

In this example, we considered the output to be composed of two parts while in the first example we used four components of the output. The procedure for both examples was identical but the labor involved was not, and in general, it increases with an increase in the number of samples in the sampling pattern.

We would also like to add a remark about the stability requirements of these systems. It has been shown in the three examples that if there are v samples per period of the sampling pattern, then the z-transform of the output is a function of z^v. If we let $z^v=\bar{z}$, then the stability requirement is simply that roots of the resulting polynomial in $\bar{z}$ lie within the unit circle in the $\bar{z}$-plane. This is the same requirement which a constant sampling rate sampled-data system must satisfy to be stable.

Conclusions

The analysis of sampled-data systems which have a periodically time varying sampling rate has been accomplished using the z-transform. When viewed at sampling instants which correspond to integer multiples of the number of samples per period of the sampling pattern, the system appears to have a constant sampling frequency, and one can obtain a closed form for the output of the system at these sampling instants. By repeating the process the proper number of times, the complete solution can be obtained. The z-transform is used in finding the solution and also serves to define the stability of the system. The only restriction is that the sampling pattern must repeat itself after a finite number of sampling intervals.

Appendix

A Short Review of the Solution of Difference Equations with Periodic Coefficients[11]

Consider the two following difference equations where a_n, b_n, c_n, and d_n are periodic coefficients of period N and P_n and Q_n are known for all n. $(n \geq 0)$

$$X_{n+1} = a_nX_n + b_nY_n + P_n \quad (28)$$

$$Y_{n+1} = c_nX_n + d_nY_n + Q_n. \quad (29)$$

A solution for these two equations can be obtained as follows. P_n and Q_n are divided into N parts. P_{KN}, $P_{KN+1}, \cdots, P_{(K+1)N-1}$, $Q_{KN}, \cdots, Q_{(K+1)N-1}$. The $2N$ equations which correspond to a complete period of the coefficients are written and each equation is substituted into the following equation. This results in two equations which contain only X_{KN}, $X_{(K+1)N}$, Y_{KN}, $Y_{(K+1)N}$ and various forcing terms. The two resulting equations are z-transformed and simultaneously solved for $X_{KN}(z)$ and $Y_{KN}(z)$.

For example, let the period of the coefficients in (28) and (29) be two. Then:

$$X_{KN+1} = X_{2K+1} = a_0K_{2K} + b_0Y_{2K} + P_{2K} \quad (30)$$

$$Y_{KN+1} = Y_{2K+1} = c_0X_{2K} + d_0Y_{2K} + Q_{2K}. \quad (31)$$

Also:

$$X_{2(K+1)} = a_1K_{2K+1} + b_1Y_{2K+1} + P_{2K+1}$$

which, using (30) and (31), can be written:

$$X_{2(K+1)} = (a_1a_0 + b_1c_0)X_{2K} + (a_1b_0 + b_1d_0)Y_{2K} + a_1P_{2K} + b_1Q_{2K} + P_{2K+1}. \quad (32)$$

Similarly, it can be found that:

$$Y_{2(K+1)} = (c_1a_0 + d_1c_0)X_{2K} + (c_1b_0 + d_1d_0)Y_{2K} + c_1P_{2K} + d_1Q_{2K} + Q_{2K+1}. \quad (33)$$

The z-transform of (32) and (33) may be written:

$$z^2[X_{2K}(z) - X_0] = (a_1a_0 + b_1c_0)X_{2K}(z) + (a_1b_0 + b_1d_0)Y_{2K}(z) + a_1P_{2K}(z) + b_1Q_{2K}(z) + P_{2K+1}(z) \quad (34)$$

$$z^2[Y_{2K}(z) - Y_0] = (c_1a_0 + d_1c_0)X_{2K} + (c_1b_0 + d_1d_0)Y_{2K} + c_0P_{2K}(z) + d_1Q_{2K}(z) + Q_{2K+1}(z) \quad (35)$$

where X_0 and Y_0 are the initial values at $t=0$. Solving (34) and (35) for $X_{2K}(z)$ and $Y_{2K}(z)$ gives:

$$X_{2K}(z) = \frac{(z^2-K_4)(z^2X_0+A_2(z))+K_2(Y_0z^2+B_2(z))}{(z^2-K_1)(z^2-K_4)-K_2K_3} \quad (36)$$

$$Y_{2K}(z) = \frac{(z^2-K_1)(z^2Y_0+B_2(z))+K_3(X_0z^2+A_2(z))}{(z^2-K_1)(z^2-K_4)-K_2K_3} \quad (37)$$

where

$$K_1 = a_1a_0 + b_1c_0; \qquad K_3 = c_1a_0 + d_1c_0;$$
$$K_2 = a_1b_0 + b_1d_0; \qquad K_4 = c_1b_0 + d_1d_0$$
$$A_2(z) = a_1P_{2K}(z) + b_1Q_{2K}(z) + P_{2K+1}(z)$$
$$B_2(z) = c_1P_{2K}(z) + d_1Q_{2K} + Q_{2K+1}(z).$$

The inverse transformation of (36) and (37) will give the values of X_{2K} and Y_{2K}; (30) and (31) can then be used to find X_{2K+1} and Y_{2K+1}.

This example assumed a period of two and was computed for two difference equations. The solution for any period and any number of equations can be obtained by the obvious extension of the above outlined procedures.

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Automatic Control of Three-Dimensional Vector Quantities—Part 1*

A. S. LANGE†

Summary—**This is primarily a tutorial paper written to acquaint control engineers with mathematics pertaining to important subsystems in modern weapons and space control systems. It is intended to minimize basic design errors often made when attempting to visualize the processes involved, particularly those involving dynamics where time as well as three-dimensional space is involved.**

A vector algebra is developed using a three-element column matrix to represent a vector and a 3 by 3 transformation matrix to represent a vector operator. The vector operator thus defined can by used to transform vectors from one set of cartesian coordinates to another. This transformation matrix is developed from the principles of plane geometry, and it is shown that several of these matrices may be combined to represent rotations in three-dimensional space. Thus, kinetic and kinematic problems in three-dimensional space can be analyzed by elementary means without the use of spherical trigonometry. It is shown that the vector algebra thus defined permits three-dimensional kinetic and kinematic problems to be stated in concise form and serves to organize the formal steps required to solve such problems. In addition, it is shown that the elements of the vector matrix represent the cartesian components of a vector in three-dimensional space which can be measured or generated by suitable combinations of commercially available instruments. Thus the techniques developed in this paper form a bridge between the theory of classical mechanics and the requirements of automatic control technology necessary for the design of control systems such as occur in modern weapons systems.

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I. Introduction

The purpose of this paper is to bring to the attention of automatic control engineers certain mathematical techniques which are applicable to the design of automatic control systems with inputs, outputs, and disturbances which may be characterized as vector quantities. Such systems include coordinate converters, fire control and guidance computers, the application of gyroscopic instruments to geometric stabilization, and inertial navigation systems. Such systems form important sub-systems in modern weapons systems and require considerable analysis and understanding of three-dimensional problems in mechanics and, more particularly, in kinetics and kinematics.

One of the difficulties associated with three-dimensional problems in mechanics is the physical visualization of the processes involved. Problems in statics, where two-dimensional or three-dimensional sketches may be employed, are clumsy but not impossible to visualize. However, problems in dynamics include time as a fourth dimension, which makes physical visualization more difficult. Further, the complexity of the dynamic coupling involved in such systems introduces a syntactical difficulty of considerable magnitude, even though the physical concepts are of an elementary nature. Consequently, it is desirable to introduce a system of notation and computation such that the necessary account keeping involved in such problems is reduced to a simple routine. At the same time, the system of notation breaks the continuum of motion into a series of static two-dimensional problems, in much the same way that the motion picture camera reduces continuous motion to a series of still pictures.

A vector algebra is defined for this purpose using elementary mathematical operations, and it is shown that these simple concepts may be combined to form a powerful analytic tool. This tool can be used to analyze and design kinetic and kinematic systems of considerable complexity; further, these concepts are useful for visualizing the behavior of these systems, and furnishing insight into their operation.

The vector algebra used here is similar to the powerful and elegant mathematical ideas found in textbooks on classical mechanics. However, the approach taken here makes use of only the most elementary mathematics, most of which is familiar to every undergraduate. Further, these ideas are presented with considerable bias toward their application to the analysis of automatic control systems, which is not usually the motive of textbooks on classical mechanics. The loss in elegance which results from the use of elementary trigonometry is compensated for by the fact that the vectors thus defined lead to equations whose solutions can be mechanized with available gyroscopes, servomotors, and resolvers.

In Part 1, the position vector is defined, along with certain transformation operations which permit problems in spherical trigonometry to be expressed as a series of operations in plane trigonometry. The application of these operators to the design of coordinate converters is demonstrated. Some of the trigonometric singularities which occur in multiple gimbal systems are discussed, and techniques for their elimination are illustrated.

The angular velocity vector is defined in Part 2, and it is shown that such vectors can be used to analyze the kinematic problems of geometric stabilization systems. The design of automatic control systems for the purpose of geometric stabilization is discussed, together with the associated problems of trigonometric singularities and servo motor performance.

Some of the properties of gyroscopic instruments are examined in Part 3. Particular attention is given to the single gimbal gyro and its application to geometric stabilization. The relations between the rate gyro and the integrating rate gyro are defined, and brief comments on multiple gimbal gyros are presented.

The appendixes consider some of the properties of vector resolving instruments, and their application to coordinate converters is examined. In addition, brief notes on vector operations are discussed, including the law of Coriolis. Euler angles are considered, and the angular acceleration of a freely rotating body is developed in terms of Euler angles as expressed in body axes.

II. The Position Vector

Consider the diagram shown in Fig. 1(a). The x_1, y_1 axes define the two-dimensional space S_1, and the x_2, y_2 axes define the two-dimensional space S_2. The point P may be located in S_1 by means of the coordinates x_1, y_1, or in S_2, by x_2, y_2. Given the angle θ between x_1 and x_2, if P is located in S_1 by the coordinates x_1, y_1, then its location is S_2 is given by

$$\begin{aligned} x_2 &= x_1 \cos\theta + y_1 \sin\theta \\ y_2 &= -x_1 \sin\theta + y_1 \cos\theta. \end{aligned} \tag{1}$$

Likewise, the position of P in S_1, in terms of its location in S_2, is given by

$$\begin{aligned} x_1 &= x_2 \cos\theta - y_2 \sin\theta \\ y_1 &= x_2 \sin\theta + y_2 \cos\theta. \end{aligned} \tag{2}$$

It is convenient to write (1) in a different form as

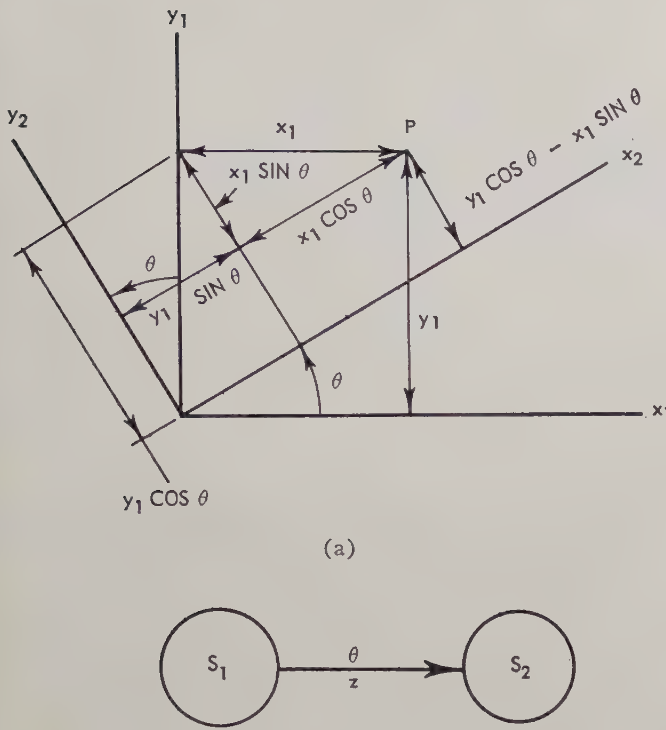
$$\begin{vmatrix} x_2 \\ y_2 \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} x_1 \\ y_1 \end{vmatrix}. \tag{3}$$

That is, (3) is (1) written in matrix form.[1] In a similar fashion, (2) may be expressed as

$$\begin{vmatrix} x_1 \\ y_1 \end{vmatrix} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} x_2 \\ y_2 \end{vmatrix}. \tag{4}$$

Several interesting comments may be made concerning these relations. First of all, (3) and (4) suggest the possibility of using the more concise matrix notation. (The desirability of a more concise notation is demonstrated in the examples below, where a succinct statement of a problem aids materially in organizing the steps needed to solve the problem.) Secondly, it may be observed that x_1, y_1 and x_2, y_2 can be thought of as the components of a position vector $\overline{R}$, locating the point P. That is,

[1] R. A. Frazer, W. J. Ducan, and A. R. Collar, "Elementary Matrices," Cambridge University Press, New York, N. Y., pp. 26–27; 1938.



(a)

(b)

Fig. 1—Plane coordinate transformation.

$$\overline{R}^1 = x_1 i_1 + y_1 j_1 = \begin{vmatrix} i_1 & j_1 \end{vmatrix} \begin{vmatrix} x_1 \\ y_1 \end{vmatrix}$$

$$\overline{R}^2 = x_2 i_2 + y_2 j_2 = \begin{vmatrix} i_2 & j_2 \end{vmatrix} \begin{vmatrix} x_2 \\ y_2 \end{vmatrix}.$$

In these equations i and j are the unit vectors along x and y, respectively, and x and y are the scalar components of the vector $\overline{R}$. Since the row matrix containing the unit vectors always prefixes the column matrix representing the components, and because the unit vectors are invariant in magnitude, the row matrices may be implied by writing

$$\overline{R}^1 = \begin{vmatrix} x_2 \\ y_1 \end{vmatrix} \qquad \overline{R}^2 = \begin{vmatrix} x_2 \\ y_2 \end{vmatrix}.$$

That is, the column matrix containing x and y may be considered as a vector quantity, subject to the usual rules of vector algebra.

The characteristics of the vector $\overline{R}$ are independent of the coordinate system in which $\overline{R}$ is expressed; the x and y components are not. Therefore, it is necessary to use the indexes "1" and "2" to identify in which space (S_1 or S_2) $\overline{R}$ is expressed. Subscripts are satisfactory for x, y, i, and j. but superscripts are used for vectors because the vector subscripts are used for other purposes, as is shown below.[2]

By using this notation, (1) and (2) may be written as

$$\overline{R}^2 = T_{21}\overline{R}^1 \tag{5}$$

$$\overline{R}^1 = T_{12}\overline{R}^2 \tag{6}$$

where

$$T_{21} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\sin\theta \end{vmatrix}$$

$$T_{12} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}.$$

The matrices T_{21} and T_{12} are called *transformation matrices* because they transform $\overline{R}^1$ to $\overline{R}^2$, and $\overline{R}^2$ to $\overline{R}^1$, respectively. It is important to stress that $\overline{R}$ itself is invariant, regardless of the space in which it is defined; the transformation matrix operates on the components of $\overline{R}$, so that the components of $\overline{R}$ in S_2 may be expressed in terms of the components of $\overline{R}$ in S_1, and vice versa.[3]

Two properties of T_{21} and T_{12} are of further interest.[4] Note that

$$T_{21}T_{12}$$

$$= \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

$$= \begin{vmatrix} \cos^2\theta + \sin^2\theta & -\sin\theta\cos\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = I$$

where I is the identity matrix. That is, T_{12} is the *inverse* of T_{21}, ($T_{12} = (T_{21})^{-1}$) since $T_{21}(T_{21})^{-1} = I$. Note also that T_{12} is the *transpose* of T_{21}, and vice versa. That is, if T_{12} is known, T_{21} may be formed by transposing the rows of T_{12} into the columns of T_{21}, etc. Matrices for which the transpose is equal to the inverse are said to be *orthogonal* matrices.

The discussion so far has been limited to two-dimensional vectors. This is an unnecessary restriction which has been used thus far merely to facilitate the discussion. If, in Fig. 1(a), the z axis is assumed to form an orthogonal set with x and y, common to both S_1 and S_2, then

$$\overline{R}^1 = x_1 i_1 + y_1 j_1 + z_1 k_1 = \begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix}$$

$$\overline{R}^2 = x_2 i_2 + y_2 j_2 + z_2 k_2 = \begin{vmatrix} x_2 \\ y_2 \\ z_2 \end{vmatrix}$$

[2] E. Weiss, "Kinematic and Geometric Relations Associated with Motion of a Vehicle over the Earth," Instrumentation Lab. Rep. No. 68 (with supplement), M.I.T., Cambridge, Mass., March, 1954.

[3] H. Goldstein, "Classical Mechanics," Addison-Wesley Co., Reading, Mass., pp. 97–101; 1950.

[4] R. A. Frazer, *et al.*, *op. cit.*, p. 3. Also, H. Goldstein, *op. cit.*, pp. 101–106.

and, as before

$$\overline{R}^2 = T_{21}\overline{R}^1$$

$$\overline{R}^1 = T_{12}\overline{R}^2.$$

where

$$T_{21} = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$T_{12} = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

Because the rotation θ is about the z axis, common to both S_1 and S_2, $z_1 \equiv z_2$. This identity is illustrated by the value of the lower right hand corner element of the transformation matrix, which is unity. Note also that for $\theta \rightarrow 0$, $T_{21} \rightarrow T_{12} \rightarrow I$. The physical meaning of this mathematical relation is that as S_2 is rotated into coincidence with S_1, $x_1 \rightarrow x_2$, $y_1 \rightarrow y_2$ and $\overline{R}^1 \equiv \overline{R}^2$.

The transformation matrix is a square matrix of third order, with three rows and three columns. According to the rules of matrix algebra, the product of a 3 by 3 square matrix and 3-element column matrix is a different 3-element column matrix,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{vmatrix} d_1 \\ d_2 \\ d_3 \end{vmatrix} = \begin{vmatrix} a_1d_1 + a_2d_2 + a_3d_3 \\ b_1d_1 + b_2d_2 + b_3d_3 \\ c_1d_1 + c_2d_2 + c_3d_3 \end{vmatrix} = \begin{vmatrix} e_1 \\ e_2 \\ e_3 \end{vmatrix}.$$

For our purposes, we say that the product of a transformation matrix and a vector in one coordinate system represents the *same* vector expressed in another coordinate system.

The rotation about the z axis represented by T_{12} and T_{21} can be suggested graphically, as shown in Fig. 1(b). Fig. 1(b) represents a "space flow diagram,"[5] which may be thought of as a short-hand way of indicating the geometry shown in Fig. 1(a). The S_1 and the S_2 in the circles of Fig. 1(b) represent the $(xyz)_1$ space and the $(xyz)_2$ space, respectively. The symbol above the connecting line between the circles, θ, represents the angle through which S_1 is rotated to obtain S_2. The arrow indicates the sense of positive rotation, following the right-hand rule,[6] and the symbol below the line, z, represents the axis about which the rotation occurs. Since, as will be demonstrated, the orientation of any two sets of cartesian coordinates can be related through a series of two-dimensional rotations like that illustrated in Fig. 1, the space flow diagram is a convenient method for representing three-dimensional diagrams.

[5] J. L. Baker, unpublished note.

[6] That is, if the thumb of the right hand is directed along the z axis, the curl of the fingers indicates the positive rotation of θ.

III. Coordinate Converters

Coordinate converters are special purpose computers which automatically solve equations in spherical trigonometry; they are used whenever it is necessary to transform data from one set of coordinates to another. For a given problem, it is possible to determine the equations describing the desired data transformation by the usual methods of spherical trigonometry. Computers can then be devised which will solve these equations. However, it is of interest to solve problems of this nature using only plane trigonometry by making use of the position vector and transformation matrixes described above. The advantages of this approach may be listed as follows:

1) It makes use of familiar relations of plane trigonometry to determine relations in spherical trigonometry which are less well known.
2) The analytic process to be described suggests to the designer the functions and thus the arrangements of the required computing components.
3) The analytic process suggests the automatic error reduction characteristic of the computing servomechanisms.
4) Items 2) and 3) combine to indicate to the automatic control designer how the necessary physical components must be interconnected in order to realize physically the automatic coordinate converter.

A typical application which requires the use of a coordinate converter is the problem of designating the position of a target with respect to some reference frame to a tracking radar mounted on some moving platform, such as a ship or airplane. It is sufficient here to assume that the target is fixed with respect to the reference frame. The following paragraphs describe this problem and the use of position vectors and transformation matrices in the solution of the problem.

Consider the Line-of-Sight (LS) from the origin of some coordinate set to a fixed point or target. This point may be located by the line-of-sight position vector $\overline{R}_{LS}$. Consider, also, the Tracking Line (TL) from the same origin to a nearby point, located by the position vector $\overline{R}_{TL}$. The tracking line might represent the optical axis of a telescope, or the axis of a radar antenna. It is desired that the telescope or radar antenna be directed toward the target, or the point designated by $\overline{R}_{LS}$. That is, it is desired that the point located by $\overline{R}_{TL}$ be coincident with that designated by $\overline{R}_{LS}$. If the telescope or antenna is mounted on a platform which can rotate about the common origin, it is necessary to compute signals with a coordinate converter which will move the telescope or the antenna with respect to the platform in such a way that $\overline{R}_{TL} = \overline{R}_{LS}$. That is, so that the tracking line and the line-of-sight always coincide.

In order to equate the two vectors $\overline{R}_{TL}$ and $\overline{R}_{LS}$ in a manner which is physically meaningful, it is necessary to express them in components which are in the same set of cartesian coordinates. It is shown below that a series of cartesian coordinate sets can be defined in such a way that two adjacent sets differ only by a single plane angle, as shown in Fig. 1. Then, if a given vector quantity is successively translated from one set of coordinates to another, it can be expressed in components of any desired set. Further, since each planar rotation of a vector can be realized by the resolver instruments described in Part 3, Appendix, it is possible to synthesize a computer or coordinate converter which works on the same principle of successive rotations.

In order to illustrate the design of such a coordinate converter, consider the following definitions, typical of fire control systems which require coordinate converters.[7]

Let the target be located in space by the angles E and B and range, where B (bearing angle) is measured in the horizontal plane from the bow of the platform, and E (elevation angle) is measured about an axis in the horizontal plane. Let the instantaneous attitude of the platform be given by *Eio* (pitch angle) and *Zo* (roll angle). *Eio* is measured about an axis in the horizontal plane, and *Zo* is measured about the longitudinal axis of the platform. Further, let the telescope or antenna be positioned with respect to the platform by two angles, Bd' (antenna train angle) and Ed' (antenna elevation angle), where Bd' is about an axis normal to the deck of the platform and Ed' is about an axis in the deck of the platform. Fig. 2(a) is the space flow diagram which describes this problem, and Fig. 2(b) is a three-dimensional sketch of the problem. Fig. 2(a) contains virtually all the information in a way which is useful to the design of the coordinate converter. The S's in Fig. 2(a) represent sets of cartesian coordinates which may be defined as:

S_i: A reference set of coordinates, with the $(xy)_i$ plane horizontal, and the z_i axis directed vertically down.

S_h: An auxiliary set of coordinates which results from a rotation B about z_i. $z_i \equiv z_h$ and the plane $(xy)_i$ is coincident with the plane $(xy)_h$. The $(xz)_h$ plane, like the $(xz)_i$ plane, is vertical and contains R_{LS} and hence, the target.

S_t: Target coordinates resulting from a rotation through the angle E about y_h so that $y_h \equiv y_t$. x_t is directed toward the target, so that $\overline{R}_{LS}$ is along x_i, and z_t forms an orthogonal set.

[7] For the most part, the notation used here for angles follows "Standard Fire Control Symbols," Bur. Ord. Publication OP-1700, Dept. of the Navy, except that the sense of all angles follows the right hand rule. This example and the method presented here is discussed in detail in W. M. Cady, M. B. Karelitz, and L. A. Turner, "Radar Scanners and Radomes," *Rad. Lab. Ser.*, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 26, pp. 123–126 and pp. 463–473; 1953. However, because of the considerable utility of this scheme, and to obtain a unified treatment, it is reported here.

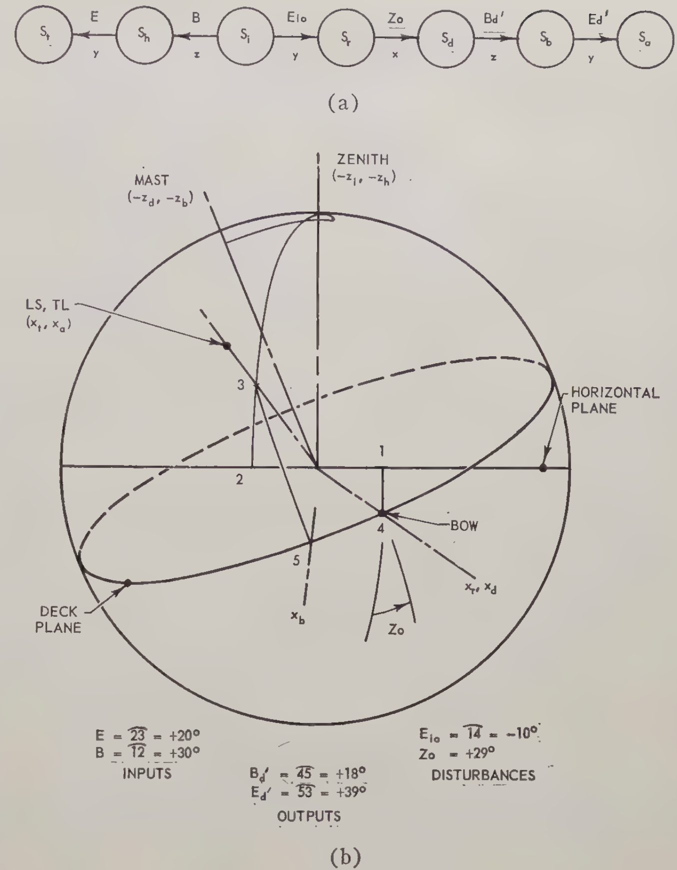


Fig. 2—(a) Space flow diagram. (b) Coordinate converter geometry.

S_r: An auxiliary set which results from the rotation through the angle *Eio* about y_i so that $y_i \equiv y_r$. The longitudinal axis of the platform is x_r. The $(xz)_r$ plane is vertical, and y_r is horizontal.

S_d: Platform coordinates which result from the rotation *Zo* about x_r so that $(xy)_d$ defines the deck of the platform; if the platform is thought of as a ship, then the mast is directed along the negative z_d axis.

S_b: A set of coordinates fixed to the outer gimbal, or train member. S_b results from the rotation Bd' about z_d so that $z_d \equiv z_b$, and the $(yx)_b$ plane is coincident with the $(xy)_d$ plane.

S_a: A set of coordinates fixed to the inner gimbal or elevation member resulting from the rotation Ed' about the y_b axis so that $y_b \equiv y_a$. The vector $\overline{R}_{TL}$ is directed along the x_a axis and is therefore contained in the $(xz)_a$ plane.

It follows from these definitions that

$$\overset{t}{\overline{R}}_{LS} = R \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \qquad \overset{a}{\overline{R}}_{TL} = R \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

where R is the distance to the target. The superscript t

on the vector $\overline{R}_{LS}$ indicates that the vector's components are expressed in S_t, target coordinates. Since, by definition, xt is directed along the line-of-sight toward the target it follows that the x component of $\overline{R}^t_{LS}$ is R, and the y and z components are zero. Or, as the vector is written, the distance to the target R is treated as a scalar quantity, and the unit value of the x component shows that the vector $\overline{R}_{LS}$ is directed along the x_t axis. In a similar way, the superscript a on the vector R_{TL} shows that that vector's components are expressed in S_a, the coordinate system defined by the antenna. It follows from the definition of S_a that the x component of the tracking line vector is unity, while the y and z components are zero.

If the two vectors $\overline{R}_{TL}$ and $\overline{R}_{LS}$ are equal, then the tracking line and the line-of-sight are coincident. But to equate them, they must both be expressed in the same coordinate set. Therefore, if the tracking line and the line of sight are to coincide

$$\overline{R}^a_{LS} = RT_{at}\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = \overline{R}^a_{TL} = R\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \tag{9}$$

where T_{at} is the transformation matrix transforming $\overline{R}^t_{LS}$ to $\overline{R}^a_{LS}$ and, from the previous description of the coordinate systems

$$T_{at} = T_{ab}T_{bd}T_{dr} \cdots T_{ht}. \tag{10}$$

Note that there is a transformation matrix associated with each line connecting the circles of the space flow diagram in Fig. 2(a). The various terms in (10) may be expanded as:

$$T_{ht} = \begin{vmatrix} \cos E & 0 & \sin E \\ 0 & 1 & 0 \\ -\sin E & 0 & \cos E \end{vmatrix}$$

$$T_{ih} = \begin{vmatrix} \cos B & -\sin B & 0 \\ \sin B & \cos B & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$T_{ri} = \begin{vmatrix} \cos Eio & 0 & -\sin Eio \\ 0 & 1 & 0 \\ \sin Eio & 0 & \cos Eio \end{vmatrix}$$

$$T_{dr} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos Zo & \sin Zo \\ 0 & -\sin Zo & \cos Zo \end{vmatrix}$$

$$T_{bd} = \begin{vmatrix} \cos Bd' & \sin Bd' & 0 \\ -\sin Bd' & \cos Bd' & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$T_{ab} = \begin{vmatrix} \cos Ed' & 0 & -\sin Ed' \\ 0 & 1 & 0 \\ \sin Ed' & 0 & \cos Ed' \end{vmatrix}.$$

Note that the principal diagonal of each transformation matrix (that is, the diagonal made up of the element in the first row and column, the element in the second row and column, and the element in the third row and column) is always composed of two cosine terms and a unity term. The remaining terms are zeros or sine terms, the zeros occurring in the row and column containing the unity term and the sine terms occurring in the rows and columns containing the cosine terms. In each case the location of the unit factor in the transformation matrix indicates the axis about which the rotation occurs. For example, the T_{dr} matrix transforms a vector from S_r to S_d about the x axis [see Fig. 2(a)] through the angle Zo, so that the upper left element is unity, and the other elements of the first row and column are zero. The remaining elements are sines or cosines of Zo. The cosine terms are located along the principal diagonal, and the sine terms are off this diagonal, so that as $Zo \to 0$, $T_{dr} \to I$.

Fig. 3(a) shows two sketches which can be used to determine the sign of the sine terms. Taking the T_{bd} matrix as an example, the two sets $(xyz)_d$ and $(xyz)_b$ are sketched in either two or three dimensions. They are displaced in this case by the angle Bd' about the common z_d, z_b axis. Thus the unity term occurs in the lower corner of the principal diagonal. From the sketches, it can be seen that for a point in the xy plane, the x_b coordinate is larger than the x_d coordinate, so that the top, or x row of T_{bd} has all positive signs. Likewise from the sketch, y_b is less than y_d, so that the sine term of the second or y row of T_{bd} is negative.

Referring again to (9), if the tracking line and the line-of-sight are to coincide

$$\overline{R}^a_{LS} = \overline{R}^a_{TL} \quad \text{or} \quad \overline{R}T_{at}\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = R\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

which requires that

$$T_{at} = \begin{vmatrix} 1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & c_2 & c_3 \end{vmatrix} \tag{11}$$

where T_{at} is a function of E, B, Eio, Zo, Bd', and Ed'. Because of the way T_{at} is defined, it is possible to show that $a_2 = a_3 = 0$. As is shown in the Appendix, (11) can be used to determine solutions for Bd' and Ed' in terms of the inputs E, B and the disturbances Eio, Zo.[8] However, it is more practical to transform R_{LS} through one space at a time, for the following reasons:

[8] Eio and Zo are termed "disturbances" because, as will be shown, if they are zero, $Ed' = E$ and $Bd' = B$. Likewise, E and B are termed "inputs" because they locate the target position with respect to the reference space S_i. Ed' and Bd' are referred to as the "output" quantities since they are the quantities to be solved for by the coordinate converter, in order to direct the antenna toward the target.

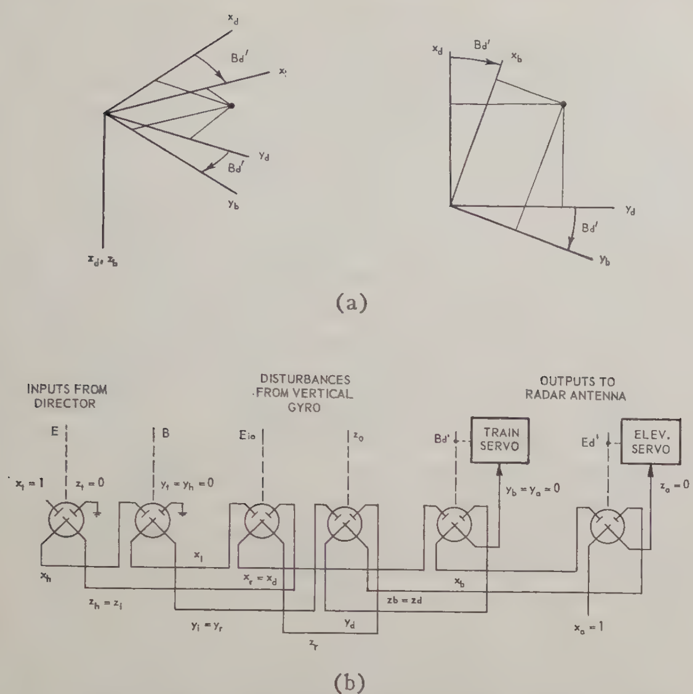


Fig. 3—Coordinate converter schematic. [See Fig. 2(b).]

1) There is less labor involved in combining a 3 by 3 matrix with a column matrix n times than there is in combining n 3 by 3 matrices since the product of the 3 by 3 transformation matrix and the three-element column matrix is always a three-element column matrix, whereas the product of two 3 by 3 matrices is another 3 by 3 matrix, the elements of which become more and more cumbersome with each transformation.
2) Each operation can be checked by a two-dimensional sketch, helping to preserve the physical significance of the operation.
3) The effect of the conditions of restraint are more clearly seen, as is shown below, by working backwards, from S_a, the inner gimbal space, towards S_d, the platform space.

Consider first the evaluation of $\overline{R}_{LS}$ in S_d, which is achieved by successive applications of the matrices T_{ht}, T_{ti}, etc., *viz*:

$$\overline{R}_{LS}^{h} = T_{ht}\overline{R}_{LS}^{t} = R\begin{vmatrix} \cos E \\ 0 \\ -\sin E \end{vmatrix}$$

$$\overline{R}_{LS}^{i} = T_{ih}\overline{R}_{LS}^{h} = R\begin{vmatrix} \cos B \cos E \\ \sin B \cos E \\ -\sin E \end{vmatrix}$$

$$\overline{R}_{LS}^{d} = T_{ri}\overline{R}_{LS}^{r} = R\begin{vmatrix} \cos B \cos E \cos Eio + \sin E \sin Eio \\ \sin B \cos E \\ \cos B \cos E \sin Eio - \sin E \cos Eio \end{vmatrix}$$

$$\overline{R}_{LS}^{d} = T_{dr}\overline{R}_{LS}^{r}$$

$$= R\begin{vmatrix} \cos B \cos E \cos Eio + \sin E \sin Eio \\ \sin B \cos E \cos Zo + (\cos B \cos E \sin Eio \\ \qquad -\sin E \cos Eio) \sin Zo \\ -\sin B \cos E \sin Zo + (\cos B \cos E \sin Eio \\ \qquad -\sin E \cos Eio) \cos Zo \end{vmatrix}. \tag{12}$$

Let $\overline{R}_{LS}{}^{d}$ be expressed by

$$\overline{R}_{LS}^{d} = R\begin{vmatrix} y_d \\ y_d \\ z_d \end{vmatrix} \tag{13}$$

so that $\overline{R}_{LS}^{b}$ may be written

$$\overline{R}_{LS}^{b} = T_{bd}\overline{R}_{LS}^{d}$$

$$= R\begin{vmatrix} x_d \cos Bd' + y_d \sin Bd' \\ -x_d \sin Bd' + y_d \cos Bd' \\ z_d \end{vmatrix} = R\begin{vmatrix} x_b \\ y_b \\ z_b \end{vmatrix} \tag{14}$$

and it follows that

$$\overline{R}_{LS}^{a} = T_{ab}\overline{R}_{LS}^{b}$$

$$= R\begin{vmatrix} x_b \cos Ed' - z_b \sin Ed' \\ y_b \\ x_b \sin Ed' + z_b \cos Ed' \end{vmatrix} = R\begin{vmatrix} x_a \\ y_a \\ z_a \end{vmatrix}. \tag{15}$$

Now consider the effect of the physical restraints on (15). Because the rotation Ed' is about the y_b axis, $y_a = y_b$, as shown by the T_{ab} matrix, which has the unity term in the y row. Now, we desire that

$$\overline{R}_{LS}{}^{a} = \overline{R}_{TL}{}^{a} = R\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix},$$

so it follows that $y_a \equiv 0$. But if $y_a = 0$, then $y_b = 0$, since $y_a = y_b$, and from (14), if $y_b = 0$,

$$Bd' = \tan^{-1}\frac{y_d}{x_d}. \tag{16}$$

Substituting for y_d and x_d from the definitions implied by (12) and (13)

$$Bd' = \tan^{-1}\frac{\sin B \cos E \cos Zo + (\cos B \cos E \sin Eio - \sin E \cos Eio) \sin Zo}{\cos B \cos E \cos Eio + \sin E \sin Eio}. \tag{17}$$

Also from (15), the condition that $z_a \equiv 0$ determines the relation

$$x_b = -z_b \cot Ed' \tag{18}$$

and the condition that $x_a = 1.0$, together with (18) gives the relation

$$-z_b \cot Ed' \cos Ed' - z_b \sin Ed' = 1.0 \tag{19}$$

or

$$Ed' = -\sin^{-1} z_b. \tag{20}$$

But because of restraints given by the definitions of the coordinate sets S_b and S_d, $z_b = z_d$. Therefore, substituting from (12)

$$Ed' = -\sin^{-1}[-\sin B \cos E \sin Zo + (\cos B \cos E \sin Eio - \sin E \cos Eio) \cos Zo]. \tag{21}$$

It may be observed that for $Zo = Eio = 0$ (*i.e.*, the disturbances are zero), (17) reduces to

$$Bd' = \tan^{-1} \tan B = B$$

and (21) becomes

$$Ed' = -\sin^{-1}(-\sin E) = E.$$

Eqs. (17) and (21) are the trigonometric expressions relating Ed' and Bd' to the inputs E and B in the presence of the disturbances Eio and Zo. It should be repeated here that these equations can be found, of course, by using the methods of spherical trigonometry. The method used here, however, makes use of plane trigonometry and some elementary properties of vectors and matrices.

The vector $\overline{R}_{LS}$ is successively transformed from one set of coordinates to another; each transformation represents an operation in plane trigonometry, which may be physically realized by electrical components such as the resolvers described in Part 3, Appendix. The restraint conditions are applied by servomechanisms, which operate on the implicit computation principle.[9] For example, see Fig. 3(b); each of the transformation matrices is instrumented by a resolver. Shaft rotations representing E, B, etc., drive these resolvers to perform the indicated transformations. The E and B rotations come from the fire control director; Eio and Zo are measured by the vertical gyro.[10,11] Bd' and Ed' are generated by servo drives. The Bd' servo is connected so that y_b is the error quantity, which is driven to zero, thus generating the correct value of Bd'. Therefore, the Bd' servo-resolver loop is an automatic computer solving (16). Likewise, the Ed' servo is connected so as to force z_a to be zero, thus solving (20) automatically. It should be emphasized that the actual electrical connections for the resolvers are indicated by the transformation matrices. The location of the unit term, for example, indicates the connection which by-passes the resolver. In the case of the T_{ih} matrix, the unit term is located in the lower left hand corner of the array indicating that the z_h component is identical to the z_i component, and thus by-passes the B resolver, as illustrated in Fig. 3(b).

When instrumenting a coordinate converter, it is sufficient to use instrument servos to drive the resolvers. Usually, however, the Ed' and Bd' servos are used to position an antenna, or a similar device in which case provision is made in the servo power gear train for an instrument gear train drive which turns the resolver at the same speed as the output shaft. In this case the coordinate converter acts as a computer for the geometric stabilization of the antenna. Operations of this kind are discussed in more detail in Part 2.

IV. Gimbal Lock

One of the difficulties associated with physical systems of which the preceding example is representative is the problem of "gimbal lock." Gimbal lock as used here is applied in a generic sense to the trigonometric singularities which arise in multiple gimbal systems. In the coordinate converter described in the previous section, for example, (17) can be used to illustrate the type of trigonometric singularity which proves troublesome in the design of coordinate converters and geometric stabilization servos, to be described in Part 2. If, in (17), we let

$$B = 0 \qquad E = \pi/2 - \epsilon$$

where ϵ is a small angle such that $\sin \epsilon \cong \epsilon$, $\cos \epsilon \cong 1$, then (17) reduces to

$$Bd' = \tan^{-1} \frac{\epsilon \sin Eio - \cos Eio}{\epsilon \cos Eio + \sin Eio} \sin Zo. \tag{22}$$

Now, for Zo and Eio small, so that the usual small angle approximations are valid

$$Bd' \cong \tan^{-1} \frac{-Zo}{\epsilon + Eio} \cdot \tag{23}$$

The physical significance of (23) may be visualized by thinking of a ship with a target almost directly overhead. Since $B = 0$, the target is forward of the mast. If the bow of the ship pitches *up* ($Eio > 0$), the mast swings *away* from the target. But if the bow pitches *down* ($Eio < 0$), the mast swings *toward* the target. In fact, for $Eio \equiv -\epsilon$, the mast points directly at the target. Now, if the ship is not rolled ($Zo \equiv 0$), the antenna remains trained on the target, regardless of the value of Eio. But for a slight amount of roll, the train member (outer gimbal) of the antenna mount must slew through 90° in order to maintain the antenna tracking line on the target. Since, in general, Zo will vary between posi-

[9] I. A. Greenwood, *et al.*, "Electronic Instruments," *Rad. Lab. Ser.*, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 21, pp. 15–21; 1948. Also, Appendix A, *infra*.

[10] L. Becker, "Gyro pickoff indications at arbitrary plane attitudes," *J. Aeronaut. Sci.*, vol. 18, pp. 718–724; November, 1951.

[11] M. J. Abzug, "Application of matrix operators to the kinematics of airplane motion," *J. Aeronaut. Sci.*, vol. 23, pp. 679–684; July, 1956.

tive and negative values, the train servo (Bd') will be commanded to "switch" from $-90°$ to zero to $+90°$. Large errors will result, whether the servo is a small instrument servo driving a resolver in a coordinate converter, or a power servo positioning on antenna. If, in addition, the target is moving slightly, it is very likely that the antenna will "lose" the target. As a general rule, the gimbal lock or trigonometric singularity occurs when the designated target is near the extension of the outer gimbal axis; the indeterminate behavior is usually eliminated in practice by the use of a redundant gimbal, although other means of alleviating this problem are discussed in Part 2 on geometric stabilization.

Consider, as another example, a fighter airplane carrying a guided missile. The airplane detects targets with a radar antenna which has a train and elevation gimbal like the antenna discussed in Section III. However, the tracking radar mounted on the guided missile may have its train axis rotated 45° from that of the fighter in order to simplify its guidance system. Therefore, if the fighter radar designates a target with a bearing of 90° and an elevation of 45°, the position of the train servo of the missile antenna will be indeterminate. This indeterminacy will be described mathematically, and it will be shown that by the use of a synthetic redundant gimbal, this indeterminacy can be eliminated.

Assume that both antenna mounts are of the train-elevation type discussed above, but that the train axis of the missile antenna is rotated through some angle α about the x_d axis. The space flow diagram is shown in Fig. 4, where

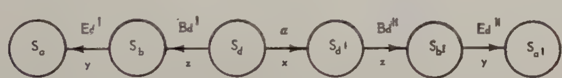


Fig. 4—Space flow diagram for slaved mount.

S_a = fighter antenna coordinates.
S_b = fighter antenna mount coordinates.
S_d = fighter coordinates.
S_d' = missile coordinates.
S_b' = missile antenna mount coordinates.
S_a' = missile antenna coordinates.
Ed' = fighter antenna elevation angle.
Bd' = fighter antenna train angle.
α = angular displacement between z_d and z_d'.
Bd'' = missile antenna train angle.
Ed'' = missile antenna elevation angle.

Now let a line-of-sight vector be defined along x_a as

$$\overline{R}_{LS}^{a} = R\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

and a tracking line vector be defined along x_a' as

$$\overline{R}_{TL}^{a'} = R\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}.$$

As before, we equate $\overline{R}_{LS}$ and $\overline{R}_{TL}$:

$$\overline{R}_{LS}^{a'} = T_{a'a}\overline{R}_{LS}^{a} = \overline{R}_{TL}^{a'} \qquad (24)$$

where

$$T_{a'a} = T_{a'b'}T_{b'd'}T_{d'd}T_{db}T_{ba} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & c_2 & c_3 \end{vmatrix}$$

and

$$T_{a'b'} = \begin{vmatrix} \cos Ed'' & 0 & -\sin Ed'' \\ 0 & 1 & 0 \\ \sin Ed'' & 0 & \cos Ed'' \end{vmatrix}$$

$$T_{b'd'} = \begin{vmatrix} \cos Bd'' & \sin Bd'' & 0 \\ -\sin Bd'' & \cos Bd'' & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$T_{d'd} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{vmatrix}$$

$$T_{ba} = \begin{vmatrix} \cos Ed' & 0 & \sin Ed' \\ 0 & 1 & 0 \\ -\sin Ed' & 0 & \cos Ed' \end{vmatrix}$$

$$T_{db} = \begin{vmatrix} \cos Bd' & -\sin Bd' & 0 \\ \sin Bd' & \cos Bd' & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

Note that T_{db} and T_{ba} are the transposes, respectively, of T_{bd} and T_{ab} given in Section III. That is, T_{db} is formed from T_{bd} by interchanging rows and columns, etc.

Proceeding as before, Bd'' is found to be

$$Bd'' = \tan^{-1}\frac{\sin Bd'\cos\alpha - \tan Ed'\sin\alpha}{\cos Bd'}. \qquad (25)$$

Now, for $Bd' = \pi/2$ and $Ed' = \pi/2 - \alpha$, (25) is mathematically indeterminate. Physically, this means that the behavior of the instrument servo used to solve (25) is indeterminate under the condition $Bd' = \pi/2$, $Ed' = \pi/2 - \alpha$. This condition occurs when the fighter antenna directs the missile antenna along the latter's outer gimbal axis. Clearly, every value of Bd'' will satisfy this condition, so that the correct value of Bd'' is ambiguous. It may be observed that this conclusion need not be limited only to the case of the rotation α about x_d. It can be demonstrated that for any case where z_b and z_b' are not aligned, there are values for Ed' and Bd' such that the resulting expression for Bd'' is indeterminate.

Now, for convenience, let $\alpha = \pi/4$. Then (25) becomes

$$Bd'' = \tan^{-1}\frac{.707}{\cos Bd'}(\sin Bd' - \tan Ed') \qquad (26)$$

which is indeterminate for $Bd' = \pi/2$, $Ed' = \pi/4$.

Fig. 5 suggests how the coordinate converter may be instrumented to eliminate this indeterminate condition,

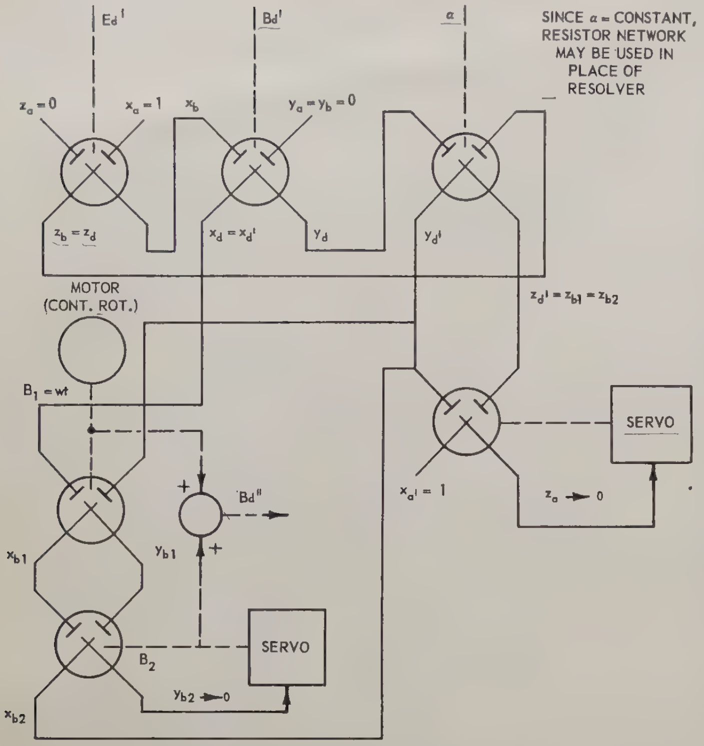


Fig. 5—Coordinate converter with anti-gimbal lock feature.

where the $B1$ resolver is driven at a contant speed w. A space flow diagram for this coordinate converter can be drawn to facilitate the derivation of (27).[12]

$$B2 = \tan^{-1} \frac{.707 \cos wt(\cos Ed' \sin Bd' - \sin Ed') - \cos Ed' \cos Bd' \sin wt}{.707 \sin wt(\cos Ed' \sin Bd' - \sin Ed') + \cos Ed' \cos Bd' \cos wt}. \qquad 27)$$

[12] This diagram resembles Fig. 4 except that S_b' is replaced by two space coordinates, S_{b_1} and S_{b_2}. Diagrammatically, circle S_d' is connected to S_{b_1}, and S_{b_1} is joined to S_{b_2} which in turn is connected to S_a' with arrows pointing right. Angle B_1 separates S_d' and S_{b_1} above the connecting line, and angle B_2 relates S_{b_1} and S_{b_2} above the arrow joining them. Since rotations through B_1 and B_2 occur about the Z axis, the z coordinate is noted below the B_1 and B_2 symbols.

It may be observed that (27) reduces to (26) when $wt = n\pi$, where $n = 0, 1, 2, \cdots$.

Dividing the numerator and denominator of (27) by $\cos Ed' \cos Bd'' \cos wt$ and substituting from (26) gives

$$B2 = \tan^{-1} \frac{\tan Bd'' - \tan B1}{1 + \tan Bd'' \tan B1}$$

$$= \tan^{-1} [\tan B_d'' - B_1)] \qquad (28)$$

where $B1 = wt$. It follows from (28) that $B2 = Bd'' - B1$, *i.e.*, $Bd'' = B1 + B2$, showing that Bd'' is in fact the sum of $B1 + B2$, as shown in Fig. 5. Bd'' is still indeterminate for the condition

$$Bd' = \pi/2 \quad Ed' = \pi/4 \quad wt = n\pi, \quad n = 0, 1, 2, \cdots,$$

but since wt has these values only momentarily, the resolver servo does not have the opportunity to develop a significant error, or experience sudden reversals, as it does if the extra resolver is not added.

The redundant resolver may be considered as a synthetic gimbal, since if two train gimbals were incorporated on the missile antenna, one of which rotated continuously, a coordinate converter similar to that shown in Fig. 5 would be required. If such were the case, however, the $B1$ resolver would be synchronized with the continuously rotating gimbal, and it would not be necessary to sum the $B1$ and $B2$ rotations, since the addition would be performed kinematically by mounting the inner train gimbal on the continuously rotating one.

On the Synthesis of Feedback Systems with Open-Loop Constraints*

JOHN A. ASELTINE†

Summary—A method is presented for synthesizing a feedback system under the constraints that the open-loop transfer function must have specified K_v and contain real poles at prescribed locations. The method is based on examination of the inverse root-locus plot for the closed-loop poles and zeros. Algebraic equations are obtained for the open-loop pole and zero locations. Examples are given for systems through fourth order in which the resulting *linear* algebraic equations are readily solved for the required compensation poles and zeros.

I. Introduction

THE feedback system synthesis problem can be stated as follows. The closed-loop transfer function defined by

$$KG_{cl}(s) = \frac{KG_{op}(s)}{1 + KG_{op}(s)} \tag{1}$$

is specified for the system of Fig. 1. The form of $G_{op}(s)$ yielding the required characteristics is to be determined.

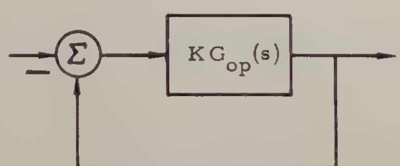


Fig. 1—Feedback system.

$G_{cl}(s)$ and $G_{op}(s)$ are assumed to be of the form

$$G_{op}(s) = \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{i=1}^{n} (s - p_i)} = \frac{s^m + \cdots}{s^n + \cdots}, \tag{2}$$

$$G_{cl}(s) = \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{i=1}^{n} (s - \pi_i)}. \tag{3}$$

If the number of poles exceeds the number of zeros, then we can write (1)

$$KG_{cl}(s) = \frac{KG_{op}(s)}{1 + KG_{op}(s)} = \frac{K(s^m + \cdots)}{(s^n + \cdots) + K(s^m + \cdots)}. \tag{4}$$

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In general, part of the open-loop transfer function belongs to the device being controlled (the so-called "plant"); the other part belongs to those elements which are needed to achieve the required $G_{cl}(s)$ performance (the compensation). That is,

$$G_{op}(s) = G_c(s)G_p(s). \tag{5}$$

The problem treated here is that of determining the form of $G_c(s)$ under the constraint imposed by $G_p(s)$. It will also be possible to specify a value of the velocity error coefficient

$$K_v = \lim_{s \to 0} sKG_{op}(s) \tag{6}$$

which is a measure of the steady-state error when the input is a ramp function.

A number of approaches have been taken to the synthesis problem. Walters[1] has shown that lead networks can be selected by simple angular measurements to cause a root locus to pass through any single desired pole location of $G_{cl}(s)$. Povejsil and Fuchs[2] expand the right-hand side of (1) after postulating the general form of compensating terms in $G_{op}(s)$. Coefficients of s are equated to determine $G_c(s)$ parameters. Aseltine[3] has shown that an inverse root-locus plot based on the poles and zeros of $G_{cl}(s)$ will locate the poles of $G_{op}(s)$. Zaborszky[4] suggests an iteration process, alternating between direct and inverse root-locus plots with small changes in $G_{op}(s)$ or $G_{cl}(s)$ made at each step. In the end, a compromise may be reached which provides the poles of $G_p(s)$ in the open loop, and at the same time a $G_{cl}(s)$ which meets specifications on transient response, bandwidth, etc. The Zaborszky method allows complex poles in $G_{op}(s)$—a definite requirement in many situations (*e.g.*, missile altitude control). One is faced when using his method, however, with a difficult (even with the help of a high-speed digital computer) iteration process of nebulous convergence properties.

Truxal[5] describes a procedure in which the starting point is specification of system characteristics. A $G_{cl}(s)$

[1] L. G. Walters, "Optimum lead-controller synthesis in feedback-control systems," IRE Trans. on Circuit Theory, vol. CT-1, pp. 45–48; March, 1954.

[2] D. J. Povejsil and A. M. Fuchs, "A method for the preliminary synthesis of a complex multiloop control system," *Trans. AIEE*, pt. 2, vol. 74, pp. 129–134; July, 1955.

[3] J. A. Aseltine, "Feedback system synthesis by the inverse root-locus method," 1956 IRE Convention Record, vol. 4, pt. 2, pp. 13–17.

[4] J. Zaborszky, "Integrated s-plane synthesis using two-way root locus," *Trans. AIEE*, pt. 1, no. 28, pp. 797–801; January, 1957.

[5] J. G. Truxal, "Automatic Feedback Control System Synthesis," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 5; 1955.

is found which will meet the specifications, and from the $G_{cl}(s)$, a $G_{op}(s)$ with real-axis poles is determined graphically. In general, the $G_{op}(s)$ so determined will not contain the poles of $G_p(s)$ (except possibly for a $G_{op}(s)$ pole at the origin). The compensating transfer function cancels the poles of $G_p(s)$ with zeros and substitutes the required $G_{op}(s)$ poles.

The synthesis problem for third-order systems has been studied recently by Hausenbauer and Lago.[6] Design curves are given relating K_v, bandwidth, and transient performance to pole and zero locations. Open-loop poles thus determined are real and do not necessarily coincide with prescribed poles. The latter are cancelled by compensation zeros as in Truxal's method.

The pros and cons of the use of $G_c(s)$ to cancel $G_p(s)$ poles are discussed by Truxal,[5] who concludes that the procedure is an acceptable one. Zaborszky[4] feels, however, that the procedure can hardly be called ideal. It would seem at least to be esthetically desirable to avoid it.

The method to be discussed in the following sections can be summarized as follows:

1) Some closed-loop poles are specified.
2) Some real open-loop poles and perhaps K_v are specified.
3) A rough inverse root locus is sketched to determine whether the specifications are consistent.
4) If the specifications are consistent, equations based on the inverse root-locus geometry are written to impose the open-loop constraints.
5) When solved, these yield pole and zero positions required by 1) and 2).

The method differs from others principally in that the open-loop constraints are a part of the procedure from the beginning. Cancellation is not necessary, so that simpler compensation can be expected. On the other hand, $G_{cl}(s)$ may contain more poles than in other methods,[4,5] these poles providing the flexibility needed to meet the constraints.

The method described here requires the poles of $G_{op}(s)$ to be real, not for reasons of RC synthesis (although this furnishes some justification for the restriction), but for geometric reasons. Segments of the real axis can always be root loci. As we shall see, the method presently makes use of this fact, and further work is needed before the real-pole requirement can be removed.

In the description of the method that follows, it is assumed that locations of critical poles of $G_{cl}(s)$ are known. These might have been obtained, for example, from design curves similar to those given by Hausenbauer and Lago[6] if the system were third order (or behaved as third order). For higher-order systems, an analog-computer study would be one means of determining acceptable $G_{cl}(s)$ pole-zero locations.

[6] C. R. Hausenbauer and G. V. Lago, "Synthesis of control systems based on approximation to a third-order system," *Applications & Industry*, vol. 39, pp. 415–421; November, 1958.

II. Open-Loop Pole Constraints

The synthesis method described here is based on the following fact: *In an inverse root locus diagram, the gain K will have the same value at each open-loop pole.* Let us illustrate this.

Suppose that we wish to achieve a closed-loop frequency response characteristic of the complex poles shown in Fig. 2. Suppose also that the open-loop transfer function has plant poles as shown. The zero has been added to provide flexibility for the constraint that the locus must pass through 0 and p_1 for the same value of gain.[7] Its location is to be determined.

In summary,

$$G_{op}(s) = \frac{s - z}{s(s - p_1)} \tag{7}$$

$$G_{cl}(s) = \frac{s - z}{(s + \alpha)^2 + \beta^2}. \tag{8}$$

An inverse root-locus plot is made as shown in Fig. 3 to test for realizability. The locus will lie along part of the real axis.

Now the gain condition for the inverse root locus can be determined by writing from (1)

$$G_{op}(s) = \frac{G_{cl}(s)}{1 - KG_{cl}(s)}. \tag{9}$$

Then, at a pole of $G_{op}(s)$,

$$|G_{cl}(s)| = \frac{1}{K}. \tag{10}$$

But at a point $s = s_1$

$$G_{cl}(s_1) = \frac{\prod [\text{distances from } s_1 \text{ to zeros of } G_{cl}(s)]}{\prod [\text{distances from } s_1 \text{ to poles of } G_{cl}(s)]}. \tag{11}$$

In the case illustrated in Fig. 2, for the pole at 0

$$\frac{1}{K} = \frac{|z|}{l_0^2} \tag{12}$$

and at p_1

$$\frac{1}{K} = \frac{|z - p_1|}{l_1^2}. \tag{13}$$

After equating these expressions, we can solve for the zero position z. This is Case I and will be discussed again in a later section.

The technique is to equate expressions for K^{-1} at each of the plant poles. A preliminary inverse root locus should be plotted with such additional compensating poles and zeros added as are necessary to make a solution feasible. The zeros, of course, will be common to $G_{cl}(s)$ and $G_{op}(s)$.

[7] If the system has more poles than zeros, the inverse and direct root-locus gains will be the same. See (4).

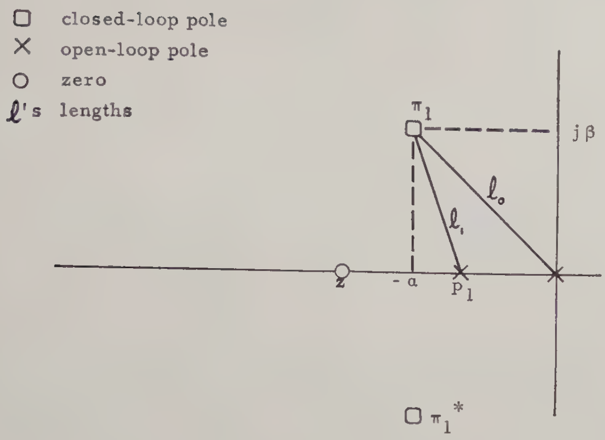

Fig. 2—Open- and closed-loop poles.

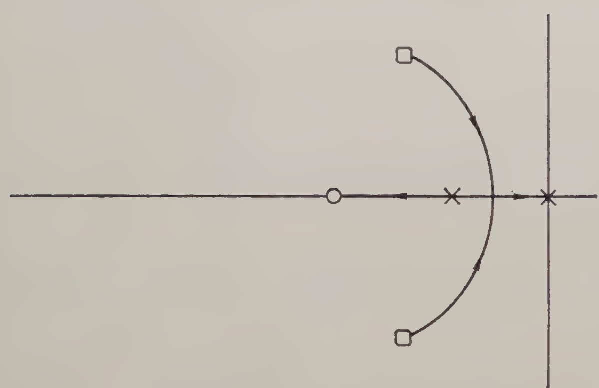

Fig. 3—Inverse root locus for verification of feasibility of configuration of Fig. 2.

III. Velocity-Constant Constraint

If we assume that one pole of $G_{op}(s)$ is at the origin, then the velocity constant K_v can be related to the poles and zeros of $G_{cl}(s)$ by the relation[8]

$$\frac{1}{K_v} = \sum_i \frac{1}{z_i} - \sum_k \frac{1}{\pi_k} \tag{14}$$

where it is assumed that $G_{cl}(s)$ is of the form

$$G_{cl}(s) = \frac{\prod (s - z_i)}{\prod (s - \pi_k)}\,. \tag{15}$$

Eq. (14) provides an additional relation among the pole-zero coordinates of the system. The use of this constraint will be illustrated in later sections.

IV. Specific Examples

Here we will consider four cases of synthesis with open-loop constraints. In each, $G_{cl}(s)$ is required to have a pair of complex poles at specified locations.

Case 1—$G_{op}(s)$ has two real poles, one at 0.

This case was started in Section II. From (12) and (13), the required zero location is given by

[8] See Truxal., *op. cit.*, pp. 281–282.

$$z = \frac{p_1}{1 - \dfrac{l_1^2}{l_0^2}}\,. \tag{16}$$

In this case, K_v is given by (14):

$$\frac{1}{K_v} = \frac{1}{z} - \frac{1}{\pi_1} - \frac{1}{\pi_1^*} = \frac{1}{z} + \frac{2\alpha}{l_0^2} \tag{17}$$

and cannot be specified independently.

Case 2—$G_{op}(s)$ has three real poles, one at 0.

The open- and closed-loop poles are shown in Fig. 4. We write

$$G_{op}(s) = \frac{s - z}{s(s - p_1)(s - p_2)}, \text{ and} \tag{18}$$

$$G_{cl}(s) = \frac{s - z}{(s - \pi_1)[(s + \alpha)^2 + \beta^2]}\,. \tag{19}$$

The zero at z and the closed-loop pole at π_1 are to be determined. The inverse root-locus plot of Fig. 5 shows the feasibility of achieving the open-loop constraints.

The gain conditions are

at 0

$$\frac{1}{K} = \frac{|z|}{l_0^2 |\pi_1|} \tag{20}$$

at p_1

$$\frac{1}{K} = \frac{|z - p_1|}{l_1^2 |\pi_1 - p_1|} \tag{21}$$

at p_2

$$\frac{1}{K} = \frac{|p_2 - z|}{l_1^2 |p_2 - \pi_1|}\,. \tag{22}$$

These are written

$$\begin{aligned} Kz - l_0^2\pi_1 &= 0 \\ Kz - l_1^2\pi_1 &= p_1(K - l_1^2) \\ Kz - l_2^2\pi_1 &= p_2(K - l_2^2). \end{aligned} \tag{23}$$

In order for there to be a solution of this set of equations, the determinant of the augmented matrix must vanish:[9]

$$\begin{vmatrix} K & -l_0^2 & 0 \\ K & -l_1^2 & p_1(K - l_1^2) \\ K & -l_2^2 & p_2(K - l_2^2) \end{vmatrix} = 0. \tag{24}$$

Eq. (24) can be solved for K:

$$K = \frac{p_2 l_2^2(l_1^2 - l_0^2) + p_1 l_1^2(l_0^2 - l_2^2)}{p_2(l_1^2 - l_0^2) + p_1(l_0^2 - l_2^2)}\,. \tag{25}$$

[9] See, for example, I. S. and E. S. Sokolnikoff, "Higher Mathematics for Engineers and Physicists," 2nd ed., McGraw-Hill Book Co., Inc., New York, N. Y., p. 118; 1941.

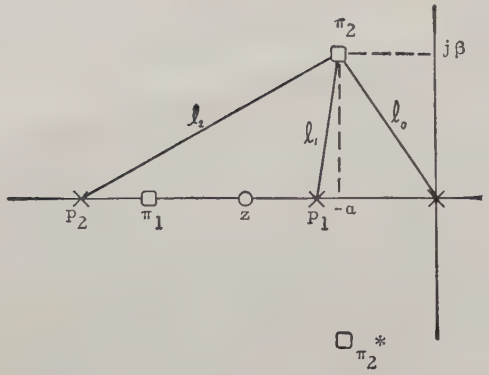

Fig. 4—Poles and zeros for Case 2.

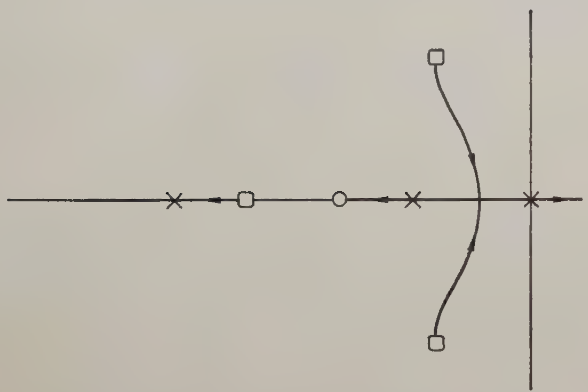

Fig. 5—Inverse root locus showing feasibility of Fig. 4.

With this value of K, any two equations of (23) can be solved for z and π_1:

$$z = \frac{p_1 l_0^2(K - l_1^2)}{K(l_0^2 - l_1^2)}, \text{ and} \tag{26}$$

$$\pi_1 = \frac{K}{l_0^2} z. \tag{27}$$

As in the previous case, K_v cannot be specified independently, *i.e.*,

$$\frac{1}{K_v} = \frac{1}{z} + \frac{2\alpha}{l_0^2} - \frac{1}{\pi_1}. \tag{28}$$

Example—Suppose we set as the following specifications:

1) poles of $G_{cl}(s)$ at $-3 \pm j4$, and
2) poles of $G_{op}(s)$ at $0, -2, -7$.

That is,

$$G_{op}(s) = \frac{s - z}{s(s + 2)(s + 7)}, \text{ and} \tag{29}$$

$$G_{cl}(z) = \frac{s - z}{(s - \pi_1)(s^2 + 6s + 25)}. \tag{30}$$

Then

$$l_1 = 5, \qquad p_1 = -2, \text{ and}$$
$$l_2 = 4.1, \qquad p_2 = -7.$$
$$l_3 = 5.65,$$

From these values we obtain

$$K = 29,$$
$$z = -2.57, \text{ and}$$
$$\pi_1 = -2.98. \tag{29}$$

The resulting direct root-locus plot is shown in Fig. 6.

Case 3—Two poles of $G_{op}(s)$ specified, one at 0; K_v specified.

Open- and closed-loop poles are shown in Fig. 7. The zero and the open-loop pole at p_2 have been added in order to achieve the flexibility needed to meet the constraints. The feasibility is demonstrated by the inverse root-locus diagram in Fig. 8. The transfer functions are

$$G_{op}(s) = \frac{s - z}{s(s - p_1)(s - p_2)}, \tag{30}$$

$$G_{cl}(s) = \frac{s - z}{(s - \pi_1)[(s + \alpha)^2 + \beta^2]}, \tag{31}$$

$$G_c(s) = \frac{s - z}{s - p_2}, \text{ and} \tag{32}$$

$$G_p(s) = \frac{1}{s(s - p_1)}. \tag{33}$$

We wish to find z and p_2.

Now the gain conditions are:

at 0,

$$\frac{1}{K} = \frac{|z|}{l_0^2|\pi_1|}; \tag{34}$$

at p_1,

$$\frac{1}{K} = \frac{|z - p_1|}{l_1^2|\pi_1 - p_1|}. \tag{35}$$

The K_v constraint is written from (14) as

$$\frac{1}{K_v} = \frac{1}{z} + \frac{2\alpha}{l_0^2} - \frac{1}{\pi_1}. \tag{36}$$

If we define

$$C \triangleq \frac{1}{K_v} - \frac{2\alpha}{l_0^2}, \tag{37}$$

we can write the three constraint equations:

$$Kz - l_0^2\pi_1 = 0.$$
$$Kz - l_1^2\pi_1 = p_1(K - l_1^2).$$
$$z - \pi_1 = -Cz\pi_1. \tag{38}$$

The third equation is nonlinear. However, if we solve the first two:

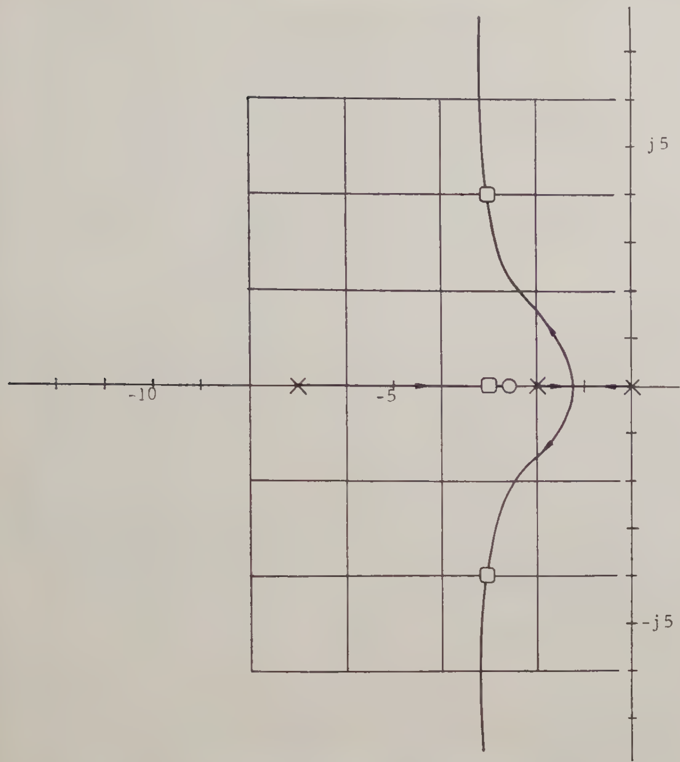

Fig. 6—Root-locus plot verifying solution of Case 2.

$$z = \frac{l_0^2 p_1(K - l_1^2)}{K(l_0^2 - l_1^2)} \text{ and}$$

$$\pi_1 = \frac{p_1(K - l_1^2)}{l_0^2 - l_1^2}, \tag{39}$$

and substitute the result in the third, the resulting equation is linear in K and yields

$$K = \frac{l_0^2[(l_0^2 - l_1^2) - l_1^2 p_1 C]}{(l_0^2 - l_1^2) - l_0^2 p_1 C}. \tag{40}$$

Example—Let us require:

1) poles of $G_{cl}(s)$ at $-3 \pm j4$,
2) poles of $G_{op}(s)$ at $0, -2$, and
3) $K_v = 10$.

That is,

$$G_{op}(s) = \frac{s - z}{s(s+2)(s - p_2)}, \tag{41}$$

$$G_{cl}(s) = \frac{s - z}{(s - \pi_1)(s^2 + 6s + 25)}, \tag{42}$$

$$G_c(s) = \frac{s - z}{s - p_2}, \text{ and} \tag{43}$$

$$G_p(s) = \frac{1}{s(s+2)}. \tag{44}$$

We are to find z and p_2. (π_1 will also be found as a consequence.)

From the specifications we write

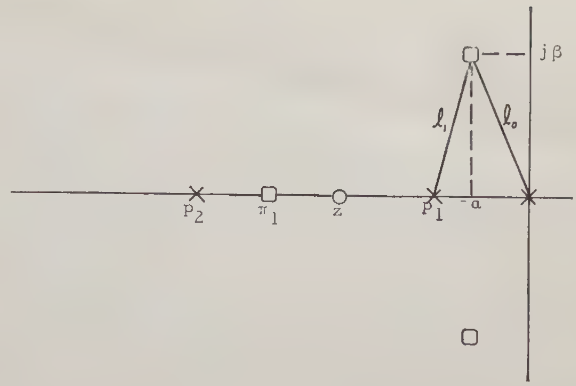

Fig. 7—Poles and zeros for Case 3.

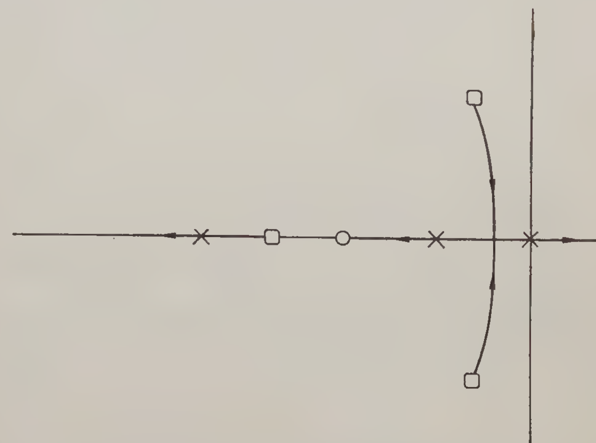

Fig. 8—Inverse root-locus test of feasibility of Case 3.

$$l_0 = 5, \qquad p_1 = -2, \text{ and}$$
$$l_1 = 4.1, \qquad C = (1/10) - (6/25)$$
$$= -0.14.$$

From these we obtain

$$K = 81.3,$$
$$z = -4.95,$$
$$\pi_1 = -16.1, \text{ and}$$
$$p_2 = -20.1. \tag{45}$$

The value of p_2 was obtained from (6) and (41) by writing

$$K_v = \frac{-Kz}{p_1 p_2}. \tag{46}$$

Case 4—$G_p(s)$ has three real poles, one at 0; K_v specified.

Open- and closed-loop poles are shown in Fig. 9. Again, a pole of $G_{op}(s)$ at p_3 and a zero at z have been added in order to meet the constraints. The transfer functions are:

$$G_{op}(s) = \frac{s - z}{s(s - p_1)(s - p_2)(s - p_3)}. \tag{47}$$

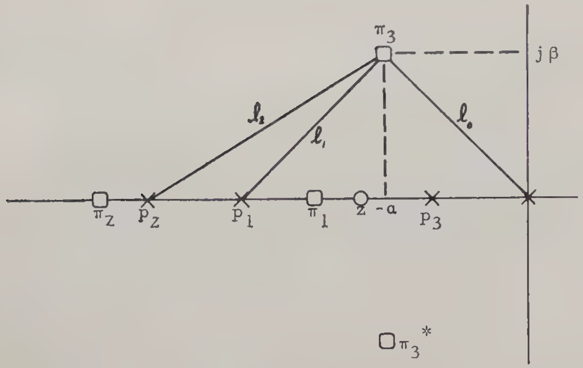

Fig. 9—Poles and zeros of Case 4.

$$G_{c1}(s) = \frac{s - z}{(s - \pi_1)(s - \pi_2)[(s + \alpha)^2 + \beta^2]} \cdot \tag{48}$$

$$G_c(s) = \frac{s - z}{s - p_3} \cdot \tag{49}$$

$$G_p(s) = \frac{1}{s(s - p_1)(s - p_2)} \cdot \tag{50}$$

We seek values of p_3 and z. (Values of π_1 and π_2 are found subsequently.)

The inverse root-locus test of consistency is shown in Fig. 10. The gain conditions are:

at 0,

$$\frac{1}{K} = \frac{|z|}{l_0^2|\pi_1\pi_2|}; \tag{51}$$

at p_1,

$$\frac{1}{K} = \frac{|p_1 - z|}{l_1^2|p_1 - \pi_1| \, |\pi_2 - p_1|}; \tag{52}$$

at p_2,

$$\frac{1}{K} = \frac{|p_2 - z|}{l_2^2|p_2 - \pi_1| \, |\pi_2 - p_2|} \cdot \tag{53}$$

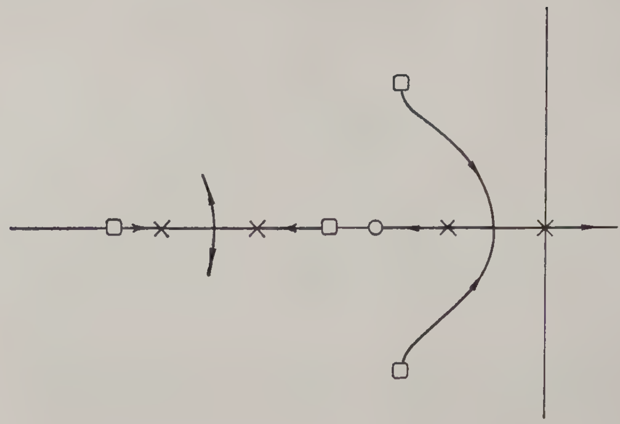

Fig. 10—Inverse root-locus test for Case 4.

where C is defined by (37). These nonlinear equations can be made linear simply by substituting the first of (55) into the others. The result is the linear set

$$\begin{aligned} K\left(1 + \frac{l_1^2}{l_0^2}\right)z - l_1^2p_1(\pi_1 + \pi_2) &= p_1K - l_1^2p_1^2 \\ K\left(1 + \frac{l_2^2}{l_0^2}\right)z - l_2^2p_2(\pi_1 + \pi_2) &= p_2K - l_2^2p_2^2 \\ \frac{KCz}{l_0^2} + (\pi_1 + \pi_2) &= \frac{K}{l^2} \cdot \end{aligned} \tag{57}$$

As in the case of (23), a solution depends on the vanishing of the determinant of the augmented matrix:

$$\begin{vmatrix} K\left(1 + \frac{l_1^2}{l_0^2}\right) & -l_1^2p_1 & p_1K - l_1^2p_1^2 \\ K\left(1 + \frac{l_2^2}{l_0^2}\right) & -l_2^2p_2 & p_2K - l_2^2p_2^2 \\ \frac{KC}{l_0^2} & 1 & \frac{K}{l_0^2} \end{vmatrix} = 0 \tag{58}$$

which determines the value of K:

$$K = \frac{l_1^2p_1^2\left(1 + \frac{l_2^2}{l_0^2}\right) - l_2^2p_2^2\left(1 + \frac{l_1^2}{l_0^2}\right) + \frac{l_1^2l_2^2}{l_0^2}p_1p_2(p_1 - p_2)C}{(p_1 - p_2)\left(1 + \frac{l_2^2}{l_0^2}\right)\left(1 + \frac{l_1^2}{l_0^2}\right) + p_1p_2\left(\frac{l_2^2 - l_1^2}{l_0^2}\right)C} \cdot \tag{59}$$

The K_v condition is

$$\frac{1}{K_v} = \frac{1}{z} + \frac{2\alpha}{l_0^2} - \frac{1}{\pi_1} - \frac{1}{\pi_2} \cdot \tag{54}$$

Rewriting (51)-(53), we have

$$\begin{aligned} Kz \qquad\qquad\qquad\qquad - l_0^2\pi_1\pi_2 &= 0 \\ Kz - l_1^2p_1(\pi_1 + \pi_2) + l_1^2\pi_1\pi_2 &= p_1K - l_1^2p_1^2 \\ Kz - l_2^2p_2(\pi_1 + \pi_2) + l_2^2\pi_1\pi_2 &= p_2K - l_2^2p_2^2. \end{aligned} \tag{55}$$

Eq. (54) is written

$$\pi_1\pi_2 - z(\pi_1 + \pi_2) = z\pi_1\pi_2C \tag{56}$$

Eq. (57) can now be solved:

$$z = \frac{p_1p_2[K(l_1^2 - l_2^2) + l_1^2l_2^2(p_1 - p_2)]}{K\left[l_1^2p_1\left(1 + \frac{l_1^2}{l_0^2}\right) - l_2^2p_2\left(1 + \frac{l_1^2}{l_0^2}\right)\right]}, \text{ and} \tag{60}$$

$$(\pi_1 + \pi_2) = \frac{K}{l_0^2}(1 - Cz). \tag{61}$$

In this case, the sum of the unspecified closed-loop poles is determined. The flexibility in choice of indi-

vidual values may be useful in adjusting, for example, transient response and bandwidth.

Linear algebraic equations also result when an additional zero is added. The equations have the form

$$\begin{aligned} a_{11}(z_1 + z_2) + a_{12}(\pi_1 + \pi_2) &= k_1 \\ a_{21}(z_1 + z_2) + a_{22}(\pi_1 + \pi_2) &= k_2. \end{aligned} \tag{62}$$

V. Conclusions

An algebraic synthesis method based on measurements on the inverse root-locus plot has been presented here. The method differs from others chiefly in that it results in an open-loop transfer function containing prescribed poles. It can further meet specification of velocity constant K_v.

As presented, the method requires that the poles of $G_{op}(s)$ be real. This restriction results from the fact that we can always make a segment of the real-axis part of the inverse root locus. Complex open-loop poles would require not only the condition that gain K be equal to the same value at each, but would also require assurance that the inverse root locus pass through the designated points. More work is needed here to remove the real-pole restriction.

The method, when applied to systems with as many as four poles, yields linear algebraic equations. The linearity will be lost for higher-order systems unless more constraints like the one on K_v are invoked. More work is needed to determine conditions leading to linear algebra for higher-order systems.

Complex-Curve Fitting*

E. C. LEVY†

Summary—**The mathematical analysis of linear dynamic systems, based on experimental test results, often requires that the frequency response of the system be fitted by an algebraic expression. The form in which this expression is usually desired is that of a ratio of two frequency-dependent polynomials.**

In this paper, a method of evaluation of the polynomial coefficients is presented. It is based on the minimization of the weighted sum of the squares of the errors between the absolute magnitudes of the actual function and the polynomial ratio, taken at various values of frequency (the independent variable).

The problem of the evaluation of the unknown coefficients is reduced to that of the numerical solution of certain determinants. The elements of these determinants are functions of the amplitude ratio and phase shift, taken at various values of frequency. This form of solution is particularly adaptable to digital computing methods, bebecause of the simplicity in the required programming. The treatment is restricted to systems which have no poles on the imaginary axis; *i.e.*, to systems having a finite, steady-state (zero frequency) magnitude.

Introduction

In the mathematical treatment of linear dynamic systems, it is usually quite advantageous to deal in the frequency domain rather than the time. In such cases, the behavior or "response" of the system to sinusoidal inputs over a band of frequencies must be known. If the dynamic system under consideration is a simple one, this characteristic of the system, or "transfer function", may be obtained analytically to a reasonable degree of accuracy. If, however, (as in most cases), the system is complex or has a large number of components, it is preferable and more reliable to define the transfer function on the basis of test results. This is done, for example, in the case of an electrical network as shown in Fig. 1, by imposing a sinusoidal voltage of known magnitude and frequency at the input end of the network, and measuring the magnitude and phase of the output voltage. Thus, for the example of Fig. 1, we could define the input and output voltages as:

$$\theta_i = |e_i| \sin(\omega t + \Phi_j) \tag{1}$$

and

$$\theta_0 = |e_0| \sin(\omega t + \Phi_0), \tag{2}$$

respectively. The transfer function of the electrical network shown, defined as the output-per-unit input would be defined by two functions, namely:

a) the amplitude ratio $E_0(\omega)/E_i(\omega)$, and

b) the phase shift $\phi_0(\omega)-\phi_i(\omega)=\Delta\phi(\omega)$, both of which vary with frequency.

In the case of the simple electrical network shown in Fig. 1, the functions $E_0(\omega)/E_i(\omega)$ and $\Delta\phi(\omega)$ can be easily obtained by established analytical methods. However, if the circuit were elaborate, experimental techniques would be found more convenient and reliable. In such case, the functions would be known in graphical form only as shown, for example, in Fig. 2.

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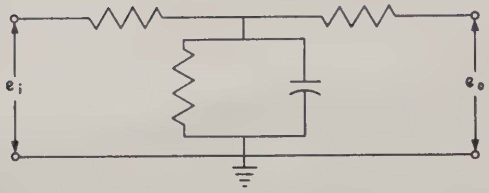

Fig. 1—Simple electrical network showing input and output terminals.

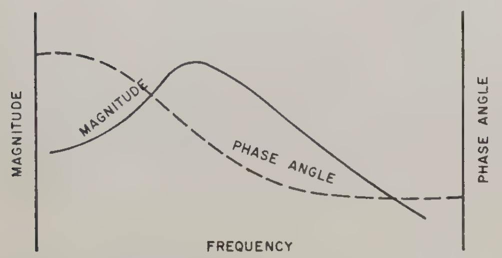

Fig. 2—Frequency response characteristics of a dynamic system.

To further the mathematical analysis of such a system, it becomes desirable to fit the curves of Fig. 2 by an algebraic expression of form suitable for further treatment. The preferred form is that of the ratio of two frequency-dependent polynomials, namely

$$G(j\omega) = \frac{A_0 + A_1(j\omega) + A_2(j\omega)^2 + A_3(j\omega)^3 + \cdots}{B_0 + B_1(j\omega) + B_2(j\omega)^2 + B_3(j\omega)^3 + \cdots}, \quad (3)$$

this form being amenable to linear transform methods of solution. In the following section, a procedure is described which leads to a $G(j\omega)$ of the above form possessing a certain minimum property.

Theory

For convenience in the manipulation of the ensuing work, (3) is rewritten in the following forms:

$$G(j\omega) = \frac{(A_0 - A_2\omega^2 + A_4\omega^4 - \cdots) + j\omega(A_1 - A_3\omega^2 + A_5\omega^4 - \cdots)}{(B_0 - B_2\omega^2 + B_4\omega^4 - \cdots) + j\omega(B_1 - B_3\omega^2 + B_5\omega^4 - \cdots)} \quad (3a)$$

$$= \frac{\alpha + j\omega\beta}{\sigma + j\omega\tau} \quad (3b)$$

$$= \frac{N(\omega)}{D(\omega)} \quad (3c)$$

with the restriction that B_0 be equal to unity.[1]

Suppose now that the function $F(j\omega)$ is used to designate the "ideal" function; *i.e.*, one which represents the data exactly. $F(j\omega)$ will then also have real and imaginary components which would coincide exactly with the values indicated by the experimental curve; *i.e.*,

$$F(j\omega) = R(\omega) + jI(\omega). \quad (4)$$

[1] This "restriction" is merely a matter of convenience. It does not affect the function in any manner; that is, it is not a restriction in the literal sense of the word.

The numerical difference between the two functions $G(j\omega)$ and $F(j\omega)$ represents the error in fitting, that is

$$\epsilon(\omega) = F(j\omega) - G(j\omega) \quad (5a)$$

$$= F(j\omega) - \frac{N(\omega)}{D(\omega)}. \quad 5b)$$

Multiplying both sides of equation (5b) by $D(\omega)$:

$$D(\omega)\epsilon(\omega) = D(\omega)F(j\omega) - N(\omega). \quad (6)$$

The right side of (6) is a function of real and imaginary terms, which may be separated to give:

$$D(\omega)\epsilon(\omega) = a(\omega) + jb(\omega) \quad (7)$$

where $a(\omega)$ and $b(\omega)$ are functions, not only of the frequency, but also of the unknown coefficients A_i and B_i. The magnitude, or absolute value of this function is:

$$|D(\omega)\epsilon(\omega)| = |a(\omega) + jb(\omega)| \quad (8a)$$

$$= \sqrt{a^2(\omega) + b^2(\omega)}. \quad (8b)$$

Then, at any specific value of frequency:

$$|D(\omega_k)\epsilon(\omega_k)|^2 = a^2(\omega_k) + b^2(\omega_k). \quad (9)$$

Let us now define E as being the function given in (9), summed over the sampling frequencies ω_k. Thus:

$$E = \sum_{k=0}^{m} [a^2(\omega_k) + b^2(\omega_k)]. \quad (10)$$

The unknown polynomial coefficients A_i and B_i are now evaluated on the basis of minimizing the function E. [It is this property which characterizes the proposed $G(j\omega)$.] To do so, we first proceed to rewrite (10) in the following form:

$$E = \sum_{k=0}^{m} [(R_k\sigma_k - \omega_k\tau_k I_k - \alpha_k)^2 + (\omega_k\tau_k R_k + \sigma_k I_k - \omega_k\beta_k)^2] \quad (11)$$

making use of (3b) and (4).

Following the accepted standard mathematical procedures, (11) is now differentiated with respect to each of the unknown coefficients A_i and B_i, and the results set equal to zero.

$$
\left.\begin{aligned}
\frac{\partial E}{\partial A_0} &= \sum_{k=0}^{m} - 2(\sigma_k R_k - \omega_k\tau_k I_k - \alpha_k) = 0 \\
\frac{\partial E}{\partial A_1} &= \sum_{k=0}^{m} - 2\omega_k(\omega_k\tau_k R_k + \sigma_k I_k - \omega_k\beta_k) = 0 \\
\frac{\partial E}{\partial A_2} &= \sum_{k=0}^{m} + 2\omega_k^2(\sigma_k R_k - \omega_k\tau_k I_k - \alpha_k) = 0 \\
\frac{\partial E}{\partial A_3} &= \sum_{k=0}^{m} + 2\omega_k^3(\omega_k\tau_k R_k + \sigma_k I_k - \omega_k\beta_k) = 0 \\
&\vdots \\
\frac{\partial E}{\partial B_1} &= \sum_{k=0}^{m} - 2\omega_k I_k(\sigma_k R_k - \omega_k\tau_k I_k - \alpha_k) + 2\omega_k R_k(\omega_k\tau_k R_k + \sigma_k I_k - \omega_k\beta_k) = 0 \\
\frac{\partial E}{\partial B_2} &= \sum_{k=0}^{m} - 2\omega_k^2 R_k(\sigma_k R_k - \omega_k\tau_k I_k - \alpha_k) - 2\omega_k^2 I_k(\omega_k\tau_k R_k + \sigma_k I_k - \omega_k\beta_k) = 0 \\
\frac{\partial E}{3 B_3} &= \sum_{k=0}^{m} + 2\omega_k^3 I_k(\sigma_k R_k - \omega_k\tau_k I_k - \alpha_k) - 2\omega_k^3 R_k(\omega_k\tau_k R_k + \sigma_k I_k - \omega_k\beta_k) = 0 \\
&\vdots
\end{aligned}\right\} \qquad (12)
$$

In the resulting equations, the terms involving the unknown coefficients may be isolated by alluding to the following linear transformations:

$$\alpha_k = A_0 - \alpha_k' \qquad (13a)$$

$$\beta_k = A_1 - \beta_k' \qquad (13b)$$

$$\left.\begin{aligned} &\sigma_k = B_0 - \sigma_k' = 1 - \sigma', \\ \text{since} \quad & \\ &B_0 = 1 \end{aligned}\right\} \qquad (13c)$$

$$\tau_k = B_1 - \tau_k'. \qquad (13d)$$

Eqs. (12) may thus be rewritten as:

$$
\left.\begin{aligned}
&\sum_{k=0}^{m} + A_0 - \alpha_k' + R_k\sigma_k' + \omega_k I_k B_1 - \omega_k I_k \tau_k' = \sum_{k=0}^{m} R_k \\
&\sum_{k=0}^{m} \omega_k^2(A_1 - \beta_k') + \omega_k I_k \sigma_k' - \omega_k^2 R_k(B_1 - \tau_k') = \sum_{k=0}^{m} \omega_k I_k \\
&\sum_{k=0}^{m} \omega_k^2 R_k \sigma_k' + \omega_k^3 I_k(B_1 - \tau_k') + \omega_k^2(A_0 - \alpha_k') = \sum_{k=0}^{m} \omega_k^2 R_k \\
&\sum_{k=0}^{m} -\omega_k^4 R_k(B_1 - \tau_k') + \omega_k^3 I_k \sigma_k' + \omega_k^4(A_1 - B_k') = \sum_{k=0}^{m} \omega_k^3 I_k \\
&\vdots \\
&\sum_{k=0}^{m} \omega_k I_k(A_0 - \alpha_k') - \omega_k^2 R_k(A_1 - \beta_k') + \omega_k^2(R_k^2 + I_k^2)(B_1 - \tau_k') = 0 \\
&\sum_{k=0}^{m} \omega_k^2 R_k(A_0 - \alpha_k') + \omega_k^3 I_k(A_1 - \beta_k') + \omega_k^2(R_k^2 + I_k^2)\sigma_k' = \sum_{k=0}^{m} \omega_k^2(R_k^2 + I_k^2) \\
&\sum_{k=0}^{m} \omega_k^3 I_k(A_0 - \alpha_k') - \omega_k^4 R_k(A_1 - \beta_k') + \omega_k^4(R_k^2 + I_k^2)(B_1 - \tau_k') = 0 \\
&\vdots
\end{aligned}\right\}. \qquad (14)
$$

Each of (14) will contain terms which are functions of the unknown coefficients, and terms which are known.

To condense the notation before expanding the above equations, the following relationships are defined:

$$\lambda_h = \sum_{k=0}^{m} \omega_k{}^h \tag{15}$$

$$S_h = \sum_{k=0}^{m} \omega_k{}^h R_k \tag{16}$$

$$T_h = \sum_{k=0}^{m} \omega_k{}^h I_k \tag{17}$$

$$U_h = \sum_{k=0}^{m} \omega_k{}^h (R_k{}^2 + I_k{}^2). \tag{18}$$

Substituting these relationships into (14) and separating the coefficients, we obtain the following set of equations:

$$\left.\begin{array}{l}
A_0\lambda_0 - A_2\lambda_2 + A_4\lambda_4 - A_6\lambda_6 + \cdots + B_1T_1 + B_2S_2 - B_3T_3 - B_4S_4 + B_5T_5 + \cdots = S_0 \\
A_1\lambda_2 - A_3\lambda_4 + A_5\lambda_6 - A_7\lambda_8 + \cdots - B_1S_2 + B_2T_3 + B_3S_4 - B_4T_5 - B_5S_6 + \cdots = T_1 \\
A_0\lambda_2 - A_2\lambda_4 + A_4\lambda_6 - A_6\lambda_8 + \cdots + B_1T_3 + B_2S_4 - B_3T_5 - B_4S_6 + B_5T_7 + \cdots = S_2 \\
A_1\lambda_4 - A_3\lambda_6 + A_5\lambda_8 - A_7\lambda_{10} + \cdots - B_1S_4 + B_2T_5 + B_3S_6 - B_4T_7 - B_5S_8 + \cdots = T_3 \\
\vdots \\
A_0T_1 - A_1S_2 - A_2T_3 + A_3S_4 + A_4T_5 - \cdots + B_1U_2 - B_3U_4 + B_5U_6 - B_7U_8 + \cdots = 0 \\
A_0S_2 + A_1T_3 - A_2S_4 - A_3T_5 + A_4S_6 + \cdots + B_2U_4 - B_4U_6 + B_6U_8 - B_8U_{10} + \cdots = U_2 \\
A_0T_3 - A_1S_4 - A_2T_5 + A_3S_6 + A_4T_7 - \cdots + B_1U_4 - B_3U_6 + B_5U_8 - B_7U_{10} + \cdots = 0 \\
\vdots
\end{array}\right\}. \tag{19}$$

Or, in matric notation:

$$(M)\cdot(N) = (C) \tag{20}$$

where

$$(M) = \left\{\begin{array}{llllllllll}
\lambda_0 & 0 & -\lambda_2 & 0 & \lambda_4 \cdots & T_1 & S_2 & -T_3 & -S_4 & T_5 \cdots \\
0 & \lambda_2 & 0 & -\lambda_4 & 0 \cdots & -S_2 & T_3 & S_4 & -T_5 & -S_6 \\
\lambda_2 & 0 & -\lambda_4 & 0 & \lambda_6 \cdots & T_3 & S_4 & -T_5 & -S_6 & T_7 \\
0 & \lambda_4 & 0 & -\lambda_6 & 0 \cdots & -S_4 & T_5 & S_6 & -T_7 & -S_8 \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
T_1 & -S_2 & -T_3 & S_4 & T_5 \cdots & U_2 & 0 & -U_4 & 0 & U_6 \cdots \\
S_2 & T_3 & -S_4 & -T_5 & S_6 \cdots & 0 & U_4 & 0 & -U_6 & 0 \\
T_3 & -S_4 & -T_5 & S_6 & T_7 \cdots & U_4 & 0 & -U_6 & 0 & U_8 \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}\right\} \tag{21a}$$

and

$$(N) = \begin{Bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ B_1 \\ B_2 \\ B_3 \\ \cdot \\ \cdot \end{Bmatrix} \quad (21b) \qquad (C) = \begin{Bmatrix} S_0 \\ T_1 \\ S_2 \\ T_3 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ U_2 \\ 0 \\ \cdot \\ \cdot \end{Bmatrix}. \quad (21c)$$

The numerical value of the unknown coefficients may thus be obtained from (20) once the matrices (21a)–(21c) have been evaluated.

Examples

Example 1) Consider the frequency response function shown in Fig. 3, representing the dynamic characteristics of an arbitrary system. The frequency function from which the curve was drawn is:

$$F(j\omega) = \frac{1 + (j\omega)}{1 + 0.1(j\omega) + 0.01(j\omega)^2}. \qquad (22)$$

Table I presents the arbitrary values selected from Fig. 3, to be used as inputs to the program.

The function chosen for the curve-fitting process is:

$$G(j\omega) = \frac{A_0 + A_1(j\omega) + A_2(j\omega)^2}{1 + B_1(j\omega) + B_2(j\omega)^2}. \qquad (23)$$

This choice is indicated by the general shape of the curves[2] presented in Fig. 3. The procedure for the numerical evaluation of the unknown coefficients is now as follows:[3] 1) Define the matrices (M), (N), and (C). Thus, for this example, they take the following form:

$$(M) = \begin{Bmatrix} \lambda_0 & 0 & -\lambda_2 & T_1 & S_2 \\ 0 & \lambda_2 & 0 & -S_2 & T_3 \\ \lambda_2 & 0 & -\lambda_4 & T_3 & S_4 \\ T_1 & -S_2 & -T_3 & U_2 & 0 \\ S_2 & T_3 & -S_4 & 0 & U_4 \end{Bmatrix},$$

$$(N) = \begin{Bmatrix} A_0 \\ A_1 \\ A_2 \\ B_1 \\ B_2 \end{Bmatrix}, \qquad (C) = \begin{Bmatrix} S_0 \\ T_1 \\ S_2 \\ 0 \\ U_2 \end{Bmatrix}.$$

[2] In most cases, the order of the polynomial expression $G(j\omega)$ can be determined from a consideration of the slopes of the magnitude curve, and the phase angle. See J. G. Truxal, "Control System Synthesis," McGraw-Hill Book Co., Inc., New York, N.Y., pp. 350–375; 1955, and G. J. Thaler and R. G. Brown, "Servomechanism Analysis," McGraw-Hill Book Co., Inc., New York, N.Y., pp. 243–249; 1953.

[3] This procedure does not have to be followed for every problem if the equations are programmed for digital computer solution.

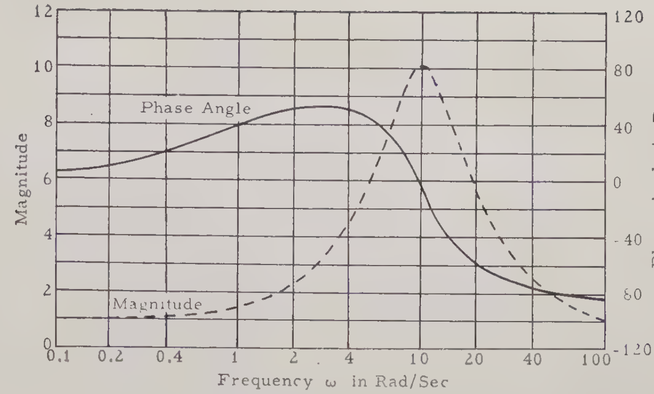

Fig. 3—Frequency response characteristics of a dynamic system with a transfer function given as:

$$F(j\omega) = \frac{1 + j\omega}{1 + 2(0.5)\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2}.$$

TABLE I

k	ω_k	magnitude	phase angle	R_k	I_k
0	0.0	1.00	0	1.00	0.000
1	0.1	1.00	5	1.00	0.090
2	0.2	1.02	10	1.00	0.177
3	0.5	1.12	24	1.02	0.450
4	0.7	1.24	31	1.05	0.630
5	1.0	1.44	39	1.10	0.900
6	2.0	2.27	51.5	1.41	1.78
7	4.0	4.44	50.5	2.82	3.42
8	7.0	8.17	28	7.23	3.82
9	10.0	10.05	− 6	10.00	−1.00
10	20.0	5.56	−59	2.85	−4.77
11	40.0	2.55	−76	0.602	−2.51
12	70.0	1.45	−82	0.188	−1.43
13	100.0	1.00	−84	0.091	−1.01

R_k = (Magnitude at ω_k) × cos (phase angle at ω_k)
I_k = (Magnitude at ω_k) × sin (phase angle at ω_k).

2) Evaluate the λ's, S's, T's, and U's. 3) Substitution in (20) gives five equations with five unknowns, which can be readily solved for each of the unknowns (A's and B's).

For this example, the numerical evaluation of the coefficient from (20) was carried out to eight significant figures, to reduce the effect of computing errors. The results thus obtained are given to five significant figures as follows:

$$A_0 = 0.99936 \qquad B_0 = 1.0000$$
$$A_1 = 1.0086 \qquad B_1 = 0.10097$$
$$A_2 = -\,0.000015983 \qquad B_2 = 0.010031.$$

By evaluating the function $G(j\omega)$ using these coefficients, it will be observed that the curve of Fig. 3 is fitted well within reading accuracy for the range $0 \leq \omega \leq 100$ rad/second, as required.

The problem of non-minimum phase systems is considered in the following example.

Example 2) Consider the frequency response function illustrated in Fig. 4. It is a graph of the function

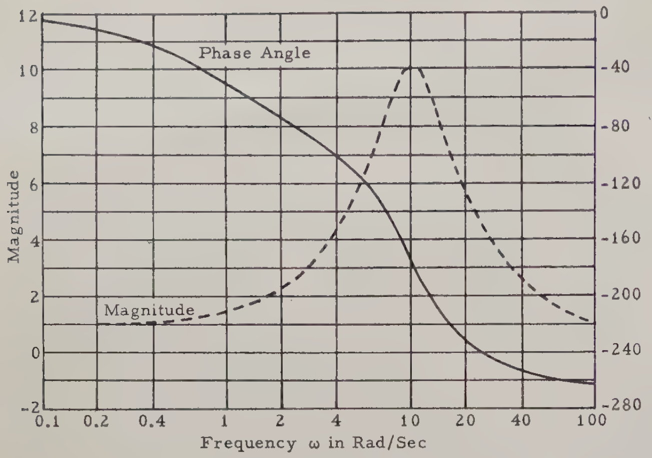

Fig. 4—Frequency response characteristics of a dynamic system with a transfer function given as:

$$F(j\omega) = \frac{1 - j\omega}{1 + 2(0.5)\frac{j\omega}{10} + \left(\frac{j\omega}{10}\right)^2}.$$

TABLE II

k	ω_k	magnitude	phase angle	R_k	I_k
0	0.0	1.00	0	1.00	0.000
1	0.1	1.00	− 6.5	1.00	−0.113
2	0.2	1.02	− 12.5	1.00	−0.220
3	0.5	1.12	− 29.5	0.975	−0.550
4	0.7	1.24	− 39.0	0.963	−0.780
5	1.0	1.44	− 51.0	0.905	−1.12
6	2.0	2.27	− 75.0	0.588	−2.20
7	4.0	4.44	−102.0	− 0.925	−4.34
8	7.0	8.17	−136.0	− 5.87	−5.69
9	10.0	10.05	−174.0	−10.00	−1.05
10	20.0	5.56	−233.5	− 3.31	4.46
11	40.0	2.55	−253.0	− 0.724	2.44
12	70.0	1.45	−261.0	− 0.227	1.43
13	100.0	1.00	−263.5	− 0.113	0.993

$$F(j\omega) = \frac{1 - j\omega}{1 + 0.1(j\omega) + 0.01(j\omega)^2}. \tag{24}$$

Table II presents the values derived from Fig. 4, and used as inputs to the digital program.

The function chosen for the curve-fitting process was the same one as before, namely that given by (23).

The numerical evaluation of the coefficients was carried out to eight significant figures, as before. The results are presented to five significant figures as follows:

$$A_0 = 0.99741 \qquad B_0 = 1.0000$$
$$A_1 = -0.99483 \qquad B_1 = 0.099607$$
$$A_2 = -0.000020400 \qquad B_2 = 0.0099847.$$

These values also represent the graph of Fig. 4 well within reading accuracy, and for a frequency range well beyond that indicated or required.

Discussion

1) Probably the most essential factor which must be realized in the application of this method is that it imposes a restriction on the types of frequency response functions that can be fitted. This restriction is such that the frequency response function must represent a system which has a finite zero frequency gain; *i.e.*, no poles at the origin. The function may, however, have zero roots; *i.e.*, zeros at the origin. The obvious alternative, if one wishes to apply this method to a function which has an infinite gain at zero frequency, is to modify the function by multiplying it by $(j\omega)^n$, n being large enough to reduce the absolute magnitude of the function at zero frequency to a finite value.

Consider, for example, the transfer function

$$F(j\omega) = \frac{1}{j\omega} \tag{25}$$

representing a pure integrator with unit gain. At $\omega = 0$, the magnitude of the function $F(j\omega)$ is undefined. If this function, or its representative graph, were multiplied by $(j\omega)^1$, a new function $F_M(j\omega)$ would be obtained, whose amplitude ratio is unity, and whose phase shift is also a constant, equal to zero degrees. The inputs to the digital computer would now be as presented in Table III.

TABLE III

k	ω_k	R_k	I_k
0	0	1	0
1	0.1	1	0
2	1	1	0
3	10	1	0
4	100	1	0

If we choose

$$G_M(j\omega) = \frac{A_0 + A_1(j\omega) + A_2(j\omega)^2}{1 + B_1(j\omega) + B_2(j\omega)^2} \tag{26}$$

as before, the results are as follows (see Appendix I):

$$A_0 = 1 \qquad B_0 = 1$$
$$A_1 = 0 \qquad B_1 = 0$$
$$A_2 = 0 \qquad B_2 = 0.$$

To obtain the representation of $F(j\omega)$, we merely divide $G_M(j\omega)$ by the same factor used to convert $F(j\omega)$ to $F_M(j\omega)$. Thus, in this case,

$$G(j\omega) = G_M(j\omega) \times \frac{1}{j\omega} = 1 \times \frac{1}{j\omega} = \frac{1}{j\omega}. \tag{27}$$

2) The method of complex-curve fitting as presented in this paper would correspond to a least-squares fit if $|D(\omega)|$ were a constant. In its indicated form, however, the method may be described as a "weighted least-squares fit," the weighting function being $|D(\omega)|^2$.

Due consideration to (8a) will show that the error $|\epsilon(\omega)|$ generally tends to assume a relative maximum when $|D(\omega)|$ is in the neighborhood of its minima. However, a local minimum in $|D(\omega)|$ corresponds to a local

maximum in $|F(j\omega)|$. This, therefore, implies that for a given value of ω, the magnitude of the error is nearly proportional to the magnitude of the function. In general, this "restriction" is not of consequence. If it is, however, the error can easily be reduced by selecting a greater number of sample points in the critical region of the curve.

3) In the process of evaluating the coefficients A_i and B_i, one of them can be assigned an arbitrary numerical value. The author chose to define the coefficient B_0 as unity. This choice, however, is not restrictive, and its selection is left to the discretion of the reader. If a different choice is made, (15) should be appropriately modified.

Appendix I

If a frequency response function such as

$$F(j\omega) = 1$$

is to be analyzed, it will be noted at the outset that the characteristic determinant is equal to zero. This leads to an indefinite solution.

A simple expedient which may be used in this case is to modify the values by some small quantity E, and then consider the limit as E approaches zero.

Thus, in this case, let

$$S_0 = \lambda_0 \qquad U_2 = \lambda_2 + E_1$$
$$S_2 = \lambda_2 \qquad U_4 = \lambda_4 + E_2$$
$$S_4 = \lambda_4 \qquad T_1 = T_3 = 0.$$

Then:

$$M = \begin{vmatrix} \lambda_0 & 0 & -\lambda_2 & 0 & \lambda_2 \\ 0 & \lambda_2 & 0 & -\lambda_2 & 0 \\ \lambda_2 & 0 & -\lambda_4 & 0 & \lambda_4 \\ 0 & -\lambda_2 & 0 & (\lambda_2 + E_1) & 0 \\ \lambda_2 & 0 & -\lambda_4 & 0 & (\lambda_4 + E_2) \end{vmatrix},$$

$$C = \begin{vmatrix} \lambda_0 \\ 0 \\ \lambda_2 \\ 0 \\ (\lambda_2 + E_1) \end{vmatrix}.$$

In terms of its minors and of the characteristic determinant, we therefore obtain:

$$A_0 = \frac{|M_{11}|}{|M|} = 1 \qquad B_0 \equiv 1$$

$$A_1 = \frac{|M_{12}|}{|M|} = 0 \qquad B_1 = \frac{|M_{14}|}{|M|} = 0$$

$$A_2 = \frac{|M_{13}|}{|M|} = \frac{E_1}{E_2} = 0 \qquad B = \frac{|M_{15}|}{|M|} = \frac{E_1}{E_2} = 0.$$

In the above, A_2 and B_2 can be made equal to zero since the magnitudes of E_1 and E_2 are arbitrary and can be assigned such values that $E_1 \ll E_2$, thus making the ratio E_1/E_2 as close to zero as need be.

Characteristics of the Human Operator in Simple Manual Control Systems*

J. I. ELKIND† AND C. D. FORGIE‡

Summary—The characteristics of simple pursuit and compensatory manual control systems were measured with a family of gaussian input signals having power-density spectra that covered a range of bandwidths, center frequencies, and some variety of shapes. The experimental results, presented in the form of graphs, show the nature of the dependence of human operator characteristics upon input-signal characteristics. The superiority of pursuit systems over compensatory systems is clearly demonstrated.

Simple analytic models that approximate these measured results are derived for both systems. The compensatory model is highly developed and relations among its parameters and those of the input have been obtained. The pursuit model is not so well developed and only approximate relations among its parameters and the input parameters have been found. The measured results and the analytic models together provide a description of manual control systems that should be useful in design of control systems.

I. Introduction

MANUAL control systems are an important class of control systems. They are in common use and have properties that are highly desirable. Particularly useful is the ability of the human operator in manual systems to modify his own characteristics so that they match the requirements of the control situation. The human operator's ability to adapt to the statistical characteristics of input signals and to the dynamics of other components of the system is well-known.[1–7]

The human operator's characteristics when functioning as a servo can be described by families of quasi-linear transfer functions and certain other functions of frequency.[6,8–10] Because the human operator behaves differently in different control situations (a control situation is a particular combination of input signal, system dynamics, and performance criteria), a single, linear, time-invariant transfer function is not sufficient to describe his characteristics. Although the operator's characteristics vary over a wide range, it appears that these variations are largely systematic and predictable. We would expect, therefore, that relations between parameters of the quasi-linear functions descriptive of human operator characteristics and parameters descriptive of the control situation could be derived. These relations would constitute adaptive analytic models for the human operator that would be useful for design of manual control systems.

This paper presents the results of experimental studies of two very simple control systems in which the dynamics of all components other than the human operator can be represented approximately by an amplifier having unity gain. Block diagrams of these systems are in Fig. 1 and Fig. 2. In both systems, the human operator's control task was as simple as it could be made. For this reason, the performance obtained is believed to be in some sense an upper bound on performance obtainable in similar systems having more complex dynamics.[3,11] The experimental data therefore provide a basic description of human operator characteristics. The statistical characteristics of the input signal, *i.e.*, amplitude, bandwidth, and shape of input power-density spectra, were the principal variables studied experimentally. To obtain high applicability of results to many practical control situations, all input signals were low-frequency Gaussian-like noise.

The principal objective of this study was derivation of analytic models for human operator characteristics that relate parameters of his characteristics to parameters of the input signal. Highly-refined models for the compensatory system (Fig. 1) and semi-quantitative

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† Bolt Beranek and Newman Inc., Cambridge, Mass.

‡ Mass. Inst. Tech., Lincoln Lab., Lexington, Mass.

[1] A. Tustin, "The nature of the operator's response in manual control and its implications for controller design," *J. IEE*, vol. 94, pt. IIA, pp. 190–202; 1947.

[2] L. Russell, "Characteristics of the Human as a Linear Servo-Element," Master's Thesis, Elec. Engrg. Dept., Mass. Inst. Tech., Cambridge; June, 1951.

[3] J. I. Elkind, "Tracking Response Characteristics of the Human Operator," Human Factors Operations Res. Labs., Air Res. and Dev. Command, USAF, Washington, D.C., Memo. 40; September, 1953.

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[6] J. I. Elkind, "Characteristics of Simple Manual Control Systems," Mass. Inst. Tech., Lincoln Lab., Lexington, Mass., Rept. 111; April, 1956, and Ph.D. dissertation, Elec. Engrg. Dept., Mass. Inst. Tech., Cambridge; May, 1956.

[7] D. T. McRuer and E. S. Krendel, "Dynamic Response of Human Operators," Wright Air Dev. Center, Wright-Patterson Air Force Base, Ohio, Tech. Rept., No. 56–524; October, 1957.

[8] M. V. Mathews, "A method for evaluating nonlinear servomechanisms," *Trans. AIEE, Applications and Industry*, vol. 74, pt. II, pp. 114–123; May, 1955.

[9] R. C. Booton, "Nonlinear Servomechanisms with Random Inputs," Dynamic Analysis and Control Lab., Mass. Inst. Tech., Cambridge, Tech. Rept. No. 70; August, 1953.

[10] W. H. Huggins, "Memo on the Experimental Determination of Transfer Functions of Human Operators and Machines," Cambridge Field Station, Air Materiel Command, Cambridge, Memo. E-4070; 1949.

[11] M. Levine, "Tracking Performance as a Function of Exponential Delay Between Control and Display," Wright Air Dev. Center, Wright-Patterson Air Force Base, Ohio, Tech. Rept. No. 53-23; October, 1953.

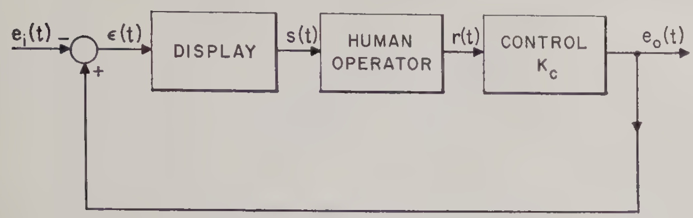


Fig. 1—Simple compensatory control system.

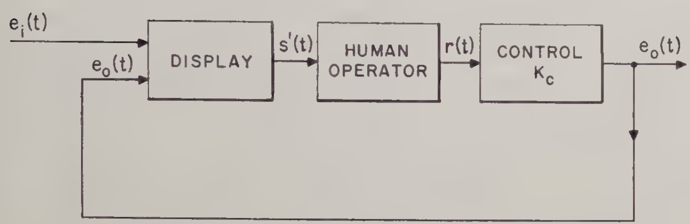


Fig. 2—Simple pursuit control system.

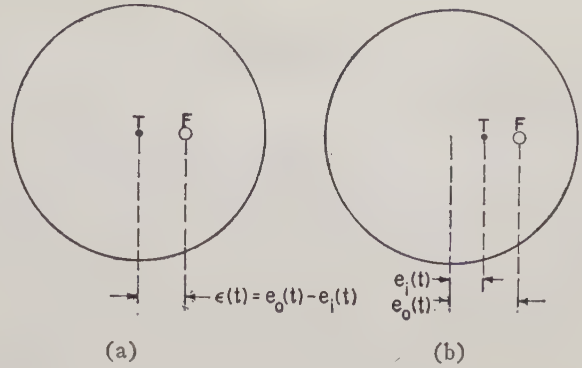


Fig. 3—Typical displays used in (a) compensatory and (b) pursuit systems. T is the target and F is the follower.

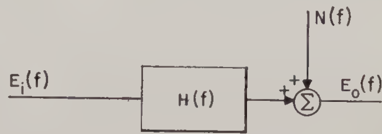


Fig. 4—Closed-loop block diagram for pursuit and compensatory systems.

models for the pursuit system (Fig. 2) were derived.

The research reported in this paper, especially the development of analytic models, is an extension of and an elaboration upon earlier work of Tustin,[1] Russell[2] and others.[4,7] Although they found analytic transfer functions that approximated human operator characteristics in certain compensatory systems, they did not derive satisfactory models relating the parameters of these transfer functions to parameters of the control situation. In the present study, such models are found for simple pursuit systems as well as for simple compensatory systems.

II. Fundamental Quantities and Relations

Although in the two systems of Fig. 1 and Fig. 2 different information is displayed to the human operator, the task objective in both systems is the same, to maintain the error or difference between input and output as close to zero as possible. In the compensatory system, only the error (difference between input and output) is presented [Fig. 3(a)]. The error is represented by the displacement of the "follower" (circle) from the stationary "target" (dot) located at the center of the display. The operator moves the control to keep the circle around the dot, thereby compensating for the error. In the pursuit system, both input and output are displayed independently [Fig. 3(b)]. The displacement of the target represents the input, and the follower, the output. The operator's task is to move the control to keep the circle around the dot, thus making follower motion correspond as closely as possible to target motion.

The closed-loop characteristics of both of these systems can be represented by the model of Fig. 4, in which it is assumed that the operator's output has two components. One component, the output of the linear transfer function $H(f)$, is linearly coherent with the input. The other component, the output of the noise generator $n(t)$, is not linearly coherent with the input. $H(f)$ is the linear transfer function with time-invariant parameters whose response to a particular input best approximates the response of the system to the same input. For stochastic signals, such as used in this study, it is most convenient to choose the mean-square-difference criterion as the basis for selecting the best approximation. The second component $n(t)$ represents those components of the response that cannot be approximated by a linear operation on the input because they result from nonlinearity, variability, and noise in the human operator's characteristics.

$H(f)$ and $n(t)$ will have invariant parameters for a particular tracking run. Parameters should vary somewhat for repeated runs with the same input signal and system dynamics because the human operator does not replicate his performance exactly. In general, a different $H(f)$ and $n(t)$ will be required to represent the system behavior when input statistics or system dynamics change. Because its parameters depend upon the control situation, $H(f)$ is called a quasi-linear transfer function. The quasi-linear transfer function alone provides an adequate representation of the manual control system when the noise power is a small fraction of the total response power. When the noise is a large fraction, both elements of the model of Fig. 4 are required.

The characteristics of $H(f)$ and $n(t)$ can be determined from measurements of system input and output through the application of statistical techniques.[12] Because the noise is not linearly coherent with the input [$\Phi_{in}(f)$ equals zero] the following relations can be derived,[5]

$$\Phi_{io}(f) = H(f)\Phi_{ii}(f) \tag{1}$$

[12] Y. W. Lee, "Applications of Statistical Methods to Communication Problems," Res. Lab. of Electronics, Mass. Inst. Tech., Cambridge, Tech. Rept. No. 181; 1950.

and

$$\Phi_{oo}(f) = |H(f)|^2\Phi_{ii}(f) + \Phi_{nn}(f) \qquad (2)$$

where $\Phi_{ii}(f)$, $\Phi_{io}(f)$, etc., are the power- or cross-power-density spectra of the signals indicated by the subscripts. $H(f)$ can be determined from (1). Φ_{nn}, the power spectrum of the noise $n(t)$, can be determined from (2) once $H(f)$ is known.

The fraction of output power that is correlated with the input,

$$1 - \frac{\Phi_{nn}(f)}{\Phi_{oo}(f)}, \qquad (3)$$

is important because it is a measure of the degree of approximation to the actual control system characteristics that is provided by the quasi-linear transfer function. The quantity $1-\Phi_{nn}/\Phi_{oo}$ will be nearly unity if $H(f)$ accounts for almost all of the operator's responses, and nearly zero if $H(f)$ accounts for only a small part of the responses.

In the compensatory system, the human operator sees only the error, and we can use the open-loop block diagram of Fig. 5 to show stimulus-response relations for the human operator. The open-loop quasi-linear transfer function $G(f)$ and noise spectrum $\Phi_{n'n'}(f)$ describe the operator's characteristics. $G(f)$ is determined by the relation

$$G(f) = \frac{H(f)}{1 - H(f)} \cdot \qquad (4)$$

In Fig. 5, the uncorrelated component of the responses is assumed, arbitrarily, to result from noise $n'(t)$ entering the system at the operator's input. The power spectrum of $n'(t)$, $\Phi_{n'n'}(f)$, can be determined from $\Phi_{nn}(f)$ and $G(f)$, at least in the region of frequency in which $G(f)$ is measured.

In the pursuit system, the human operator sees and responds to both input and error, and a more complicated open-loop block diagram like that of Fig. 6 is required to represent his characteristics. $P_i(f)$ operates on the input signal; $G_2(f)$ on the error signal. $G_1(f)$ represents the dynamics of components common to both channels.

The quantities, $P_i(f)$, $G_2(f)$, and $G_1(f)$ are quasi-linear transfer functions whose combined effect is equivalent to the closed-loop transfer function $H(f)$. However, these three elements cannot be determined uniquely from measurements only on input and output. By adding to the error signal a second input that is statistically independent of the primary input and small in amplitude, it would be possible to determine the product transfer functions, $P_iG_1(f)$ and $G_2G_1(f)$. This procedure was not attempted, but certain additional measurements and tests were made from which semi-quantitative models for the elements in the diagram of Fig. 6 were derived.

The power-density spectra from which $H(f)$ and $\Phi_{nn}(f)$ were computed were determined directly from the time functions of the input, output, and error signals. A special purpose analog computer[6] was used to compute the spectra.

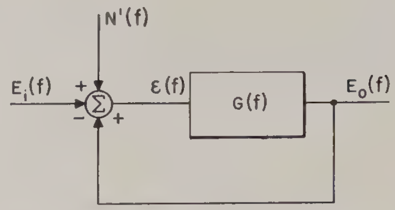


Fig. 5—Open-loop block diagram for compensatory system.

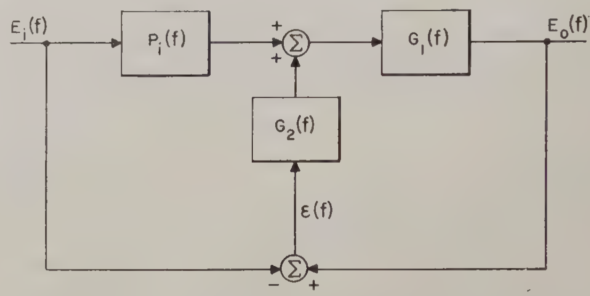


Fig. 6—Open-loop block diagram for pursuit system. The noise sources are not shown.

III. Experimental Study

Four experiments were performed to explore separately the variability in operator characteristics and the effects on operator characteristics of amplitude, bandwidth, and shape of input-power spectrum. In each experiment, the pursuit-compensatory dichotomy was one of the variables. In this paper, only the results of the last two experiments (bandwidth and shape) will be discussed in detail.

The first experiment (variability) showed that human operator characteristics were stable in that they did not change with time (after the subjects had learned the task) and that differences in performance of the several subjects were not significant. The second experiment (amplitude) showed that the human operator's characteristics do not depend strongly upon the mean-square amplitude of the input.

Throughout this study, input signals having approximately the characteristics of low-frequency Gaussian noise were used. Low-frequency Gaussian noise is an idealization of actual inputs to most manual control systems that is mathematically tractable, lends itself to precise specification, and is easy to generate. Results derived from tests with gaussian noise are likely to be more applicable to situations in which the input is aperiodic and complex than would results derived from tests with simple sinusoids or step functions.

Input signals were generated by summing a large number (usually between 40 and 144) of sinusoids of different frequencies, arbitrary phases, and uniform spacing in frequency. Producing the signals in this way

permitted very good control over the shape of the spectrum and, in particular, made it possible to achieve very sharp cutoff. A signal with 40 components looks quite random and has an amplitude distribution that is approximately normal.[13]

In Experiment III (bandwidth), the input signals had Rectangular Spectra [as illustrated in Fig. 7(a)] of various cutoff frequencies. The rms amplitude was adjusted to 1 inch, measured at the face of the display oscilloscope, for each test. Cutoff frequencies were 0.16, 0.24, 0.40, 0.64, 0.96, 1.6, 2.4, and 4.0 cps. These inputs are designated R.16, R.24, R.40, etc. Tracking is very easy with a cutoff frequency of 0.16 cps but becomes very difficult when the cutoff is greater than 1 cps.

In Experiment IV (shape), two types of inputs were used. The RC Filtered Spectra [Fig. 7(b)] simulate spectral shapes encountered in actual control situations more closely than most of the other signals. They were obtained by passing a signal having a rectangular spectrum with a 2.88 cps cutoff through one, two, and three cascaded RC filters. Each filter had a critical frequency of 0.24 cps. These three inputs are designated F1, F2, and F3, respectively. For comparison purposes, a signal (designated F4) having rectangular spectrum with a cutoff frequency of 0.24 cps was added to this group of three signals. The Selected Band Spectra [Fig. 7(c)] were produced by combining in various ways four signals having rectangular spectra of equal bandwidth but located on the frequency scale in adjacent blocks. The combinations used and their designations are shown in Fig. 7(c). All of these signals had an amplitude of 1 inch rms.

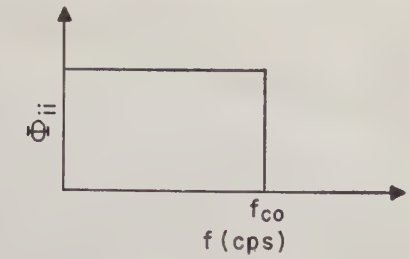
A description of the experimental apparatus and procedure is in Appendix A. A full discussion of the experimental results appears in Elkind.[6]

IV. Analytic Models for the Human Operator

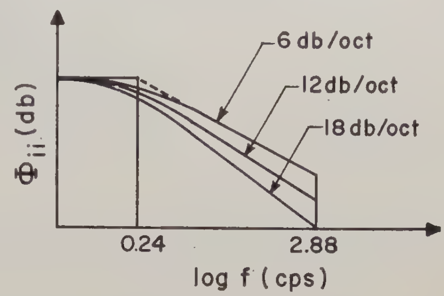
In this section, we first derive simple analytic functions that approximate the measured open-loop transfer functions $G(f)$ and the output noise spectra $\Phi_{nn}(f)$ of the compensatory system. Then we determine the relations a) among the parameters of those analytic functions, and b) between these parameters and parameters of the input signals. These are the main relations at which this study was aimed. With the pursuit system, we do not have the data necessary for derivation of as complete an analytic model. However, we are able to determine a semiquantitative model by relying upon physical reasoning to support the experimental results. This model provides fair approximation to the measured characteristics of the pursuit system.

A complete model for human operator characteristics must include representations for both the correlated part of operator response, represented by quasi-linear

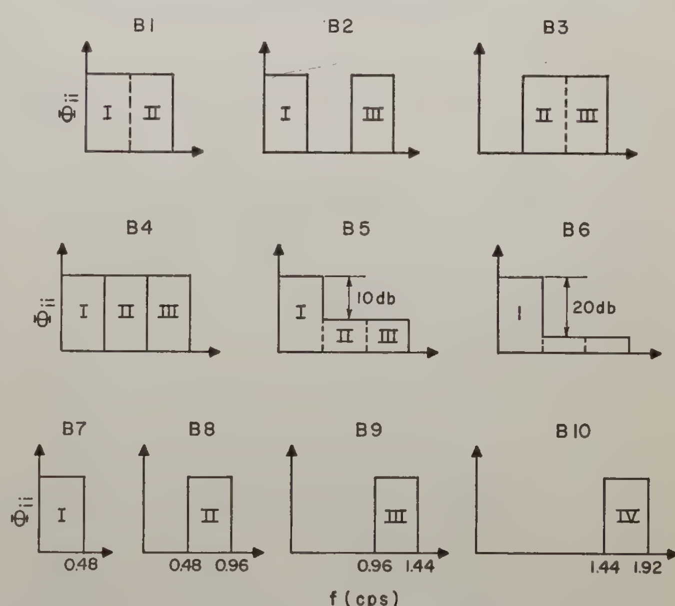
[13] S. O. Rice, "Mathematical analysis of random noise," *Bell Sys. Tech. J.*, vol. 24, pp. 46–158; January, 1945.



(a) Experiment III (bandwidth) Rectangular Spectra, R.16 through R4.0.



(b) Experiment IV (shape) RC Filtered Spectra, F1 through F4.



(c) Experiment IV (shape) Selected Band Spectra, B1 through B10.

Fig. 7—Input Spectra.

transfer functions, and the uncorrelated part, represented by noise power-density spectra. For the compensatory system, two analytic functions, $G_a'(f)$ and $G_a(f)$, are defined that provide good approximations to the measured open-loop transfer functions $G(f)$ of Fig. 5. The noise power-density spectra $\Phi_{nn}(f)$ of the closed-loop model of Fig. 4 are approximated by a quadratic function for both the compensatory and pursuit systems. Compensatory open-loop noise spectra such as $\Phi_{n'n'}(f)$ of Fig. 5 can be found from $\Phi_{nn}(f)$ and $G_a(f)$. Reasonable forms for transfer functions $P_i(f)$, $G_1(f)$, and $G_2(f)$, the elements of the open-loop pursuit block diagram of Fig. 6, are also postulated. Pursuit open-loop noise spectra can also be found from Φ_{nn} and P_i, G_1, and G_2.

A. Model for the Compensatory System

1) Open-Loop Transfer Functions: The forms of the analytic functions used to approximate $G(f)$ were suggested by the experimental results and by certain basic characteristics of visual-manual responses: 1) the human operator is fundamentally a low-pass device; his open-loop gain must go to zero at high frequencies; 2) for low-bandwidth inputs, his low-frequency gain can be very high but must remain finite because of limitations of the visual, motor, and memory systems. Thus, although the human operator can act as a low-pass filter with high gain, he cannot act as true integrator; 3) any approximation to $G(f)$ must contain a delay analogous to the reaction-time delay or stimulus-response latency of the human operator in a discrete tracking task.

a) First approximation $G_a'(f)$: The simplest transfer function that satisfies these requirements has the form

$$G_a'(f) = \frac{Ke^{-2\pi jf\alpha}}{\left(\frac{jf}{f_0} + 1\right)} \tag{5}$$

where K is the low-frequency gain, α' is the delay in seconds, and f_0 is the bandwidth in cps.[14] The symbol $G_a'(f)$ is used to represent the first analytic approximation to the measured open-loop transfer functions $G(f)$.

With certain low-bandwidth inputs, the parameters determined by fitting (5) to the measured open-loop characteristics led to instability in the closed loop. Apparently the delay α' obtained by fitting (5) is too large, and negative phase margin at gain-crossover frequency results. Since the actual control system was stable, the analytic transfer functions determined from (5) cannot be completely correct. However, in all cases of instability, the gain-crossover frequency of $G_a'(f)$ was in a region in which the input power was very small and in which $G(f)$ could not be measured. The difficulty evidently is that the approximation to the low-frequency portion of $G(f)$ provided by $G_a'(f)$ is not valid at high frequencies.

b) Second approximation $G_a(f)$: The high-frequency behavior of $G(f)$ cannot be determined from measurements restricted to low frequencies. Analytic transfer functions that are stable and that have the same low-frequency characteristics as (5) can, however, be postulated. A low-pass filter with high critical frequencies produces nearly linear phase lag and almost no attenuation (acts like a pure delay) at very low frequencies. The low-frequency characteristics of such a low-pass filter can be approximated by a single low-pass RC filter.[15] Thus, a second simple analytic model for $G(f)$ is

$$G_a(f) = \frac{Ke^{-2\pi jf\alpha}}{\left(\frac{jf}{f_0} + 1\right)\left(\frac{jf}{f_1} + 1\right)} \tag{6}$$

where values for K and f_0 obtained by fitting (6) to measured transfer functions are the same as those obtained by fitting (5), f_1 is greater than f_0, and α is a new value for the delay [$1/2\pi f_1$ seconds less than α' of (5)]. The behavior of (6) can be stabilized by a proper choice of f_1.

Unique values for α and for f_1 of (6) cannot be determined from the low-frequency part of measured $G(f)$. However, the greatest values that satisfy the requirement of stability can be specified. These maximum values, denoted by the symbols $\alpha_{\max}$ and $f_{\max}$, are the values of α and f_1 corresponding to the $G_a(f)$ that has zero phase margin. The results of Experiment IV showed that the human operator reduces his phase margin when the high-frequency content of the input is reduced; it is likely that the actual phase margin, if it could be measured, would be close to zero. The use of the parameters $\alpha_{\max}$ and $f_{\max}$ to describe the measured characteristics therefore seems reasonable.

Analytic functions of the forms (5) and (6) were fitted to the open-loop characteristics of the compensatory system for all the runs of Experiments III and IV. Before the fitting was done, the bandpass characteristics (B3, B8, B9, and B10) were translated down the frequency scale, the low-frequency cutoffs being set at zero. Values obtained for some of the parameters of $G_a'(f)$ and $G_a(f)$ are shown in Table I below. The analytic functions were fitted visually to the measured characteristics by one author (JIE) and were checked and corrected by the second author (CDF). The authors believe that the values chosen for the parameters are probably correct to within the following tolerances: ± 3 db for K, $\pm 0.2\, f_0$ cps for f_0, and ± 0.02 second for $\alpha_{\max}$.

2) Behavior of the Parameters of the Analytic Functions

a) Delay $\alpha_{\max}$: The mean value of the delay $\alpha_{\max}$ (Table I) obtained with low-pass input signals (all inputs except bandpass inputs B3, B8, B9, and B10) is 0.13 second. The standard deviation of the values of $\alpha_{\max}$ is 0.022 second. No consistent pattern of variation related to input-signal characteristics is apparent. We shall, therefore, treat the delay $\alpha_{\max}$ as a constant equal to the mean, 0.13 second.

The values for $\alpha_{\max}$ obtained with bandpass inputs (B3, B8, B9, and B10) are very much larger than any of the delays obtained with low-pass inputs. They were not included in the computation of the mean $\alpha_{\max}$ because with bandpass inputs the human operator appears to use a very different mode from that used with low-pass signals. He first attempts to generate a sinusoidal response synchronized with the center frequency of the input and then he tries to modulate this sinusoid to reproduce the envelope of the input. The complexity of

[14] Values for parameters obtained by fitting (5) to the measured transfer functions are in Table I and Fig. 8.

[15] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand, Inc., New York, N. Y., pp. 303–336; 1945.

TABLE I

SUMMARY OF PARAMETERS OF $G_a'(f)$ AND $G_a(f)$

Input	α_{max} (second)	f_{max} (cps)	α' (second)	f_0 (cps)	Kf_0 (cps)
R.16	0.11	0.30	0.64	0.035	1.9
R.24	0.10	0.99	0.26	0.050	1.9
R.40	0.13	2.0	0.21	0.13	1.7
R.64	0.15	4.8	0.18	0.28	1.5
R.96	0.14	∞	0.14	0.58	1.2
R1.6	0.12	∞	0.12	0.60	0.56
R2.4	0.12	∞	0.12	0.30	0.21
F1	0.14	∞	0.14	0.18	0.60
F2	0.13	∞	0.13	0.050	0.89
F3	0.18	∞	0.18	0.030	1.3
F4	0.10	0.78	0.31	0.056	2.0
B1	0.15	∞	0.15	0.76	2.1
B2	0.11	∞	0.11	0.80	0.95
B3	0.28	∞	0.28	2.0	1.8
B4	0.15	∞	0.15	2.0	1.9
B5	0.13	∞	0.13	0.30	1.1
B6	0.15	∞	0.15	0.16	1.2
B7	0.10	2.8	0.16	0.14	2.0
B8	0.22	1.0	0.39	0.50	1.4
B9	0.39*	∞	0.39*	2.0*	2.1*
B10	1.1*	∞	1.1*	0.45†	—

* Approximate.
† Lead.

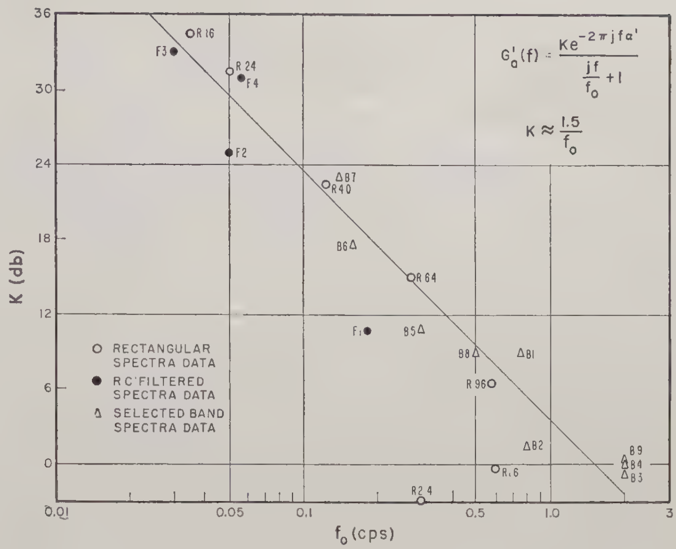


Fig. 8—Gain K vs bandwidth of $G_a'(f)$, the first analytic approximation. B10 is not plotted because its characteristics are not low-pass and cannot be approximated by $G_a'(f)$.

this mode may be the cause of the long delay.

The mean value for α_{max} obtained in this study is slightly lower than the reaction times that have been obtained with discrete visual stimuli. For the discrete case, reaction times of about 0.17 to 0.25 second are representative.[16] In a continuous task, we would expect a shorter delay, since the time at which a response must be made is less uncertain than it is in the discrete task. Because of this difference in experimental conditions, a value of 0.13 second does not seem unreasonable.

b) Gain-bandwidth product Kf_0: The plot of K in db vs log f_0 of Fig. 8 indicates that f_0, the bandwidth of $G_a'(f)$, is roughly inversely proportional to the gain K. The approximate relation is

$$K = \frac{1.5}{f_0}, \tag{7}$$

indicating that the gain-bandwidth product equals about 1.5 cps on the average.

If K is large and if f_{max} is greater than Kf_0, the magnitude of $G_a(f)$ reaches 0 db at approximately Kf_0 cps. At this frequency the phase lag must be less than 180° to satisfy stability requirements. If f_{max} is smaller than Kf_0, the magnitude of $G_a(f)$ reaches 0 db at a frequency lower than Kf_0, and therefore the phase lag at Kf_0 also must be less than 180°. If f_0 is small, the first lag of (6) contributes nearly 90° phase shift at frequency Kf_0. The phase lag contributed by the delay α must therefore be less than 90°. For α of 0.13 second, the phase lag of the delay is 90° at about 2 cps. This, then, is the maximum permissible value of Kf_0. Note that in Table I, the largest values of Kf_0 are only slightly greater than 2 cps. Kf_0 is closely related to phase margin. Table I shows that as the bandwidth or high-frequency content of the input increases, Kf_0 decreases indicating an increase in phase margin.

c) Critical frequency of second lag, f_{max}: The values for f_{max} in Table I are, with one minor exception, 10 to 20 times larger than f_0 and considerably greater than the input bandwidth. With high-bandwidth inputs or with inputs containing high-frequency components, the system is stable without the second lag and f_{max} is infinite.

d) Gain K: The family of Rectangular Spectra of Experiment III provides a most effective vehicle for discovering relations between K and input-signal parameters. The important characteristics of the Rectangular Spectra are summarized by a single parameter, the cutoff frequency f_{co}. A plot of K vs f_{co} (Fig. 9) shows that K is very nearly inversely proportional to the square of the cutoff frequency. Only at extreme values of f_{co} does K depart significantly from this relation. The approximate relation between K and f_{co} is

$$K = \frac{2.2}{f^2_{co}}. \tag{8}$$

The RC Filtered and the Selected Band Spectra cannot be described simply by cutoff frequency. More fundamental measures of input-signal characteristics which apply equally well to all the input spectra must be found. We would expect, *a priori*, that the predictability of the input and its location on the frequency scale would be important determinants of K. Clearly, K will be larger for a predictable input than for an unpredictable input. Translating a signal in frequency does not affect its predictability, but the results with the band-pass inputs (B7, B8, B9, and B10) show that the gain decreases as the input band is moved up the frequency

[16] R. S. Woodworth and H. Schlosberg, "Experimental Psychology," Henry Holt, and Co., Inc., New York, N. Y.; pp. 8–42; 1954.

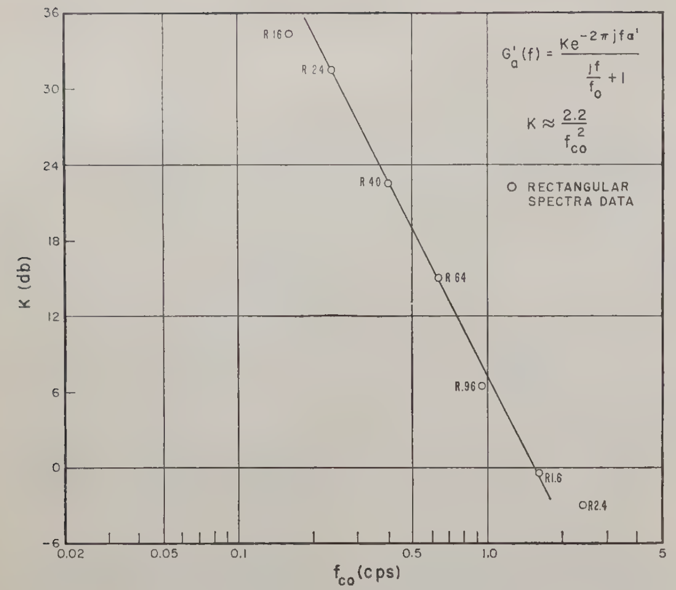


Fig. 9—Measured gain K of analytic approximations vs cutoff frequency f_{co} of Rectangular Spectra.

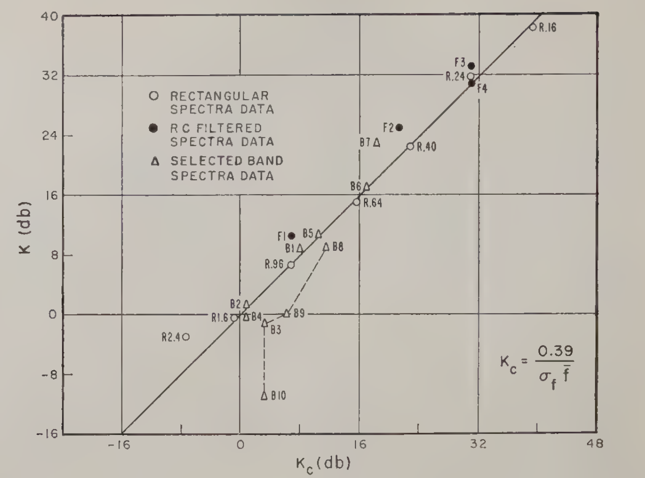


Fig. 10—Measured gain K of analytic approximations vs computed gain K_c for all Input Spectra.

scale. Thus, it is evident that location in frequency (center frequency) comes into play.

Many quantities describe in a general way the location and predictability of a signal. Expressions involving several of the most likely quantities were compared with the experimental results. It was found that one particular pair produced the best match to all the measured values of K. The members of the pair are $\bar{f}$, the mean frequency, and σ_f, the standard deviation of the spectrum. The mean frequency $\bar{f}$ is the first moment of the input power-density spectrum normalized with respect to the mean-square value of input. It is a measure of location. The standard deviation σ_f is the square root of the second moment of the spectrum about its mean, normalized in the same manner. It is a measure of spectral width and therefore of predictability. The quantities $\bar{f}$ and σ_f are defined in the standard manner by the following relations:

$$\bar{f} = \frac{\int_0^\infty f\Phi_{ii}df}{\int_0^\infty \Phi_{ii}df} \tag{9}$$

and

$$\sigma_f = \left[\frac{\int_0^\infty f^2\Phi_{ii}df}{\int_0^\infty \Phi_{ii}df} - (\bar{f})^2\right]^{1/2}. \tag{10}$$

For the Rectangular Spectra, the relation

$$K = \frac{0.39}{\bar{f}\sigma_f} \tag{11}$$

provides a good approximation to the measured values of gain K. Using (11), values for gain can be computed for the RC Filtered and Selected Band Inputs. Fig. 10 is a plot of the computed values, denoted K_c, against measured values of gain K for the complete set of input signals. Except for the bandpass inputs connected by the dashed line, computed and measured gain are in very good agreement. Omitting the bandpass data, (11) accounts for almost all the variance of K; the correlation between K and K_c (in db) is 0.986.

The bandpass data of Fig. 10 indicate that (11) apparently does not weight $\bar{f}$ heavily enough. As $\bar{f}$ increases, the difference between K and K_c also increases. Several relations between $\bar{f}$, σ_f, and K were investigated. An exponential weighting of $\bar{f}$ gave the best results. Using only the bandpass data, the relation

$$K = \frac{3.8}{\sigma_f e^{\bar{f}/0.38}} \tag{12}$$

provides a good approximation to the measured values of K.

Using (12), computed values for gain (denoted K_c') can be determined for all the other inputs. A plot of measured gain K vs K_c' (in db), Fig. 11, shows that (12) is a very good approximation to almost all the measured values of gain. The correlation between K and K_c' (in db) is 0.976, indicating that, for the entire set of input signals, (12) accounts for almost all the variance of K.

On an intuitive level, the exponential weighting of $\bar{f}$ in (12) seems more correct than the linear weighting in (11). As the mean frequency goes to zero, the exponential approaches unity, and the gain remains finite (if σ_f is not zero). As $\bar{f}$ becomes large, the exponential increases rapidly, and the gain approaches zero as should be the case.

Use of the quantities σ_f and $\bar{f}$ makes possible highly-accurate prediction of the measured gain. These predictions are good for a very wide range of different input spectra. Note that although the form and the coefficients of (11) and (12) were determined from only part of the data, the computed values of gain obtained from these relations approximate closely almost all the measured

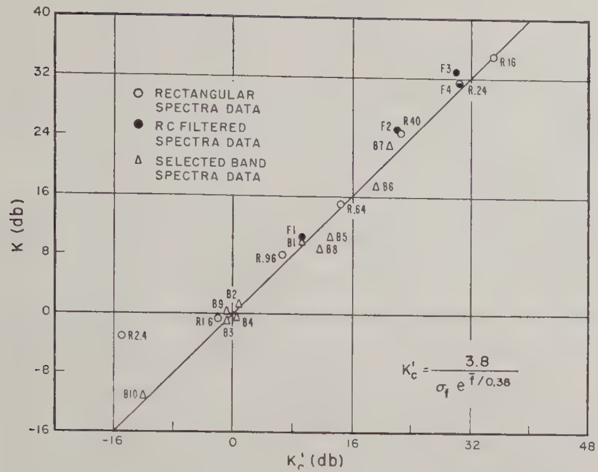


Fig. 11—Measured gain K of analytic approximations vs computed gain K_c' for all Input Spectra.

values of K. These observations confirm our belief that σ_f and $\bar{f}$ are important and basic parameters for predicting human operator characteristics.

3) Models for Compensatory Noise: Measured noise spectra can be approximated reasonably well by the quadratic function

$$\frac{\Phi_{nn}(f)}{\int_0^\infty \Phi_{ii}df} = \frac{c_n^2}{\left[\left(\frac{jf}{f_n}\right)^2 + 2\zeta\frac{jf}{f_n} + 1\right]^2} \tag{13}$$

where the parameters f_n, ζ, and c_n depend upon the input-signal characteristics. The data can be approximated by other functions, but the quadratic seems to provide the best approximation over the entire range of input signals studied.[17]

For most inputs, ζ was between 0.8 and 1.0 and f_n between 1 and 2 cps. In developing the analytic model, ζ was considered a constant. Except for the band-pass inputs, f_n is approximately equal to the phase-crossover frequency of $G_a(f)$. The error or difference between this frequency and f_n is almost always less than 20 per cent.

The magnitude c_n^2 varies over a wide range. A model for the relation between input and c_n^2, based on the hypothesis that most of the noise is produced by random errors in operator response, is given below. A second interesting model in which the noise is assumed to result from random variations in operator transfer function is given by McRuer and Krendel.[7]

Assume for simplicity the operator's responses can be approximated by a series of step-like movements spaced randomly in time.[18] Assume that each step has two components: one part, Δe_0, that is linearly correlated with the input signal and the other part that is not linearly correlated. This second part is the noise or random error in the operator's responses. Assume that the noncoherent component of each step is independent of all other steps. Assume also that the mean-square magnitude of the uncorrelated or noise component is proportional to the mean-square magnitude of Δe_0. Thus,

$$\overline{n^2} \propto (\overline{\Delta e_0})^2. \tag{14}$$

Since the mean-square magnitude of the response steps $\overline{\Delta e_0^2}$ is approximately proportional to mean-square velocity of the input and the square of the average magnitude of the closed-loop transfer function H_a, it follows[6] that,

$$c_n^2 \propto \frac{H_a^2 \int_0^\infty f^2\Phi_{ii}df}{f_n \int_0^\infty \Phi_{ii}df}. \tag{15}$$

For convenience, denote the terms on the right by c_c^2. A plot of c_n^2 vs c_c^2 (in db) for all input signals is in Fig. 12.

Fig. 12 indicates that the model for c_n^2 has approximately the correct form and accounts for most of the variance in the measured values of c_n^2. Correlation between c_n^2 and c_c^2 is 0.90. The basic assumption in the derivation of the model was that the noise spectrum Φ_{nn} results from random errors and not from system nonlinearities. The fact that this assumption leads to a model that predicts c_n^2 with reasonable accuracy indicates that such a random process may be the principal source of noise.

B. Models for the Pursuit System

1) Closed-Loop Transfer Functions: Assume that in the pursuit closed-loop block diagram of Fig. 6, $G_1(f)$ represents certain inherent characteristics that limit the operator's response capability. Limiting characteristics might include effects of such factors as nerve-impulse transmission-time delay and response time of the neuromuscular system. Assume further than $P_i(f)$ is a predictor whose function is to estimate the future of the input in order to correct for the lags and delays introduced by $G_1(f)$. $G_2(f)$ provides feedback necessary to reduce noise and error. Note that if $P_iG_1(f) = 1$, the signal part of the error is zero. In this case, $G_2(f)$ operates only on the noise and does not affect the signal.[19]

a) Inherent characteristics $G_1(f)$: An estimte of $G_1(f)$ was obtained from measurements of one subject's responses to input step displacements of known amplitude but unknown time of initiation. The subject was

[17] McRuer and Krendel[7] use $\sin^2 x/x^2$ to approximate the data obtained with rectangular input spectra. However, when the complete set of noise data are considered, the quadratic form of (13) appears to provide better over-all fits than does $\sin^2 x/x^2$.

[18] Although the operator's responses are assumed here to be discrete, his task and the visual stimuli are continuous. The argument made earlier to justify the use of a low, 0.13-second, value of delay still holds.

[19] G. Lang and J. M. Ham, "Conditional feedback systems: a new approach to feedback control," *Trans. AIEE, Applications and Industry*, vol. 74, pt. II, pp. 152–161; May, 1955.

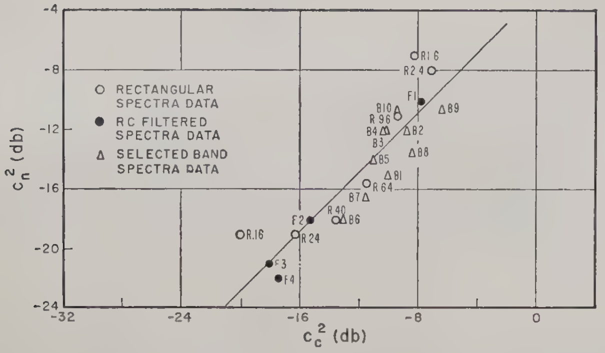


Fig. 12—c_n^2, measured magnitude of noise power-density spectrum vs c_c^2, computed magnitude for all compensatory results.

instructed to respond as rapidly as possible. A fast response to a step usually consists of several discrete movements, each of which appears not to be altered very much while it is beig made.[20] A measurement of the initial response movement should exclude most of the effects of feedback and provide an estimate of $G_1(f)$.

An average step response obtained from repeated trials with one of the subjects was approximated by the time function whose Fourier transform is

$$\frac{G_1(f)}{2\pi jf} = \frac{e^{-2\pi jf\alpha}}{2\pi jf} \frac{1}{\left(\frac{jf}{f_m}\right)^2 + 2\zeta\left(\frac{jf}{f_m}\right) + 1} \tag{16}$$

where α is 0.205 second, f_m is 4.0 cps, and ζ is 0.8. However, the delay determined in this experiment probably is not a good value for the delay in a continuous tracking situation. A better value is that used in compensatory model of (6), 0.13 second. At low frequencies, the quadratic of (16) acts like a delay of 0.064 second, and therefore, the low-frequency behavior of $G_1(f)$ can be approximated by a delay of 0.194 second.

b) Feedback $G_2(f)$: We do not have sufficient data to make quantitative statments about $G_2(f)$. However, $G_2(f)$ performs a function similar to that of the open-loop transfer function of the compensatory system. Therefore, $G_2(f)$ is likely to have a form similar to $G_a(f)$, the second analytic approximation for the compensatory system. $G_2(f)$ should, however, not contain a delay because it is included in $G_1(f)$. Because there are two channels for signal flow in the pursuit system, the requirements on $G_2(f)$ are less stringent than those on the open-loop transfer function of the compensatory system. We would expect the gain and bandwidth of $G_2(f)$ to be lower than in the compensatory system.

c) Predictor $P_i(f)$: The characteristics of $P_i(f)$ depend upon $G_1(f)$, the noise generated by the human operator, and the statistical characteristics of the input signal. We can assume that, subject to the restrictions imposed by these factors, the human operator tries to adjust $P_i(f)$ so as to minimize his tracking error. The error criterion that the human operator uses to adjust $P_i(f)$ depends upon the instructions that he is given and upon the nature of the task. The form of $P_i(f)$ and its ability to act as an optimum predictor are limited by the human operator's capacity for perceiving high derivatives of the input.

A first approximation to $P_i(f)$ can be obtained if we make the assumptions that: 1) the human operator adjusts $P_i(f)$ so that it best corrects for the lags and delays introduced by $G_1(f)$ with minimum mean-square error; 2) the human operator makes use of only the displacement and velocity of the input to predict the input; 3) the human operator does not consider his own noise when establishing the optimum $P_i(f)$; and 4) $G_2(f)$ can be neglected when computing the optimum $P_i(f)$.

These assumptions were made for the following reasons. In a simple tracking task, the mean-square-error criterion is probably a close approximation to the human operator's actual error criterion. It is also the most tractable criterion to use with random signals. The visual system can perceive displacement with good accuracy and velocity with fair accuracy, but is not well-suited to measuring acceleration or higher derivatives. The noise is neglected to simplify the computation of the optimum $P_i(f)$. If the magnitude of Φ_{nn} is small, we probably will not introduce important errors by making this simplification. Neglecting $G_2(f)$ simplifies the calculations for $P_i(f)$. If $P_iG_1(f)$ is approximately unity, $G_2(f)$ does not affect greatly the system's response to the input.

Under these assumptions

$$P_i(f) = b_0 + b_1(2\pi jf) \tag{17}$$

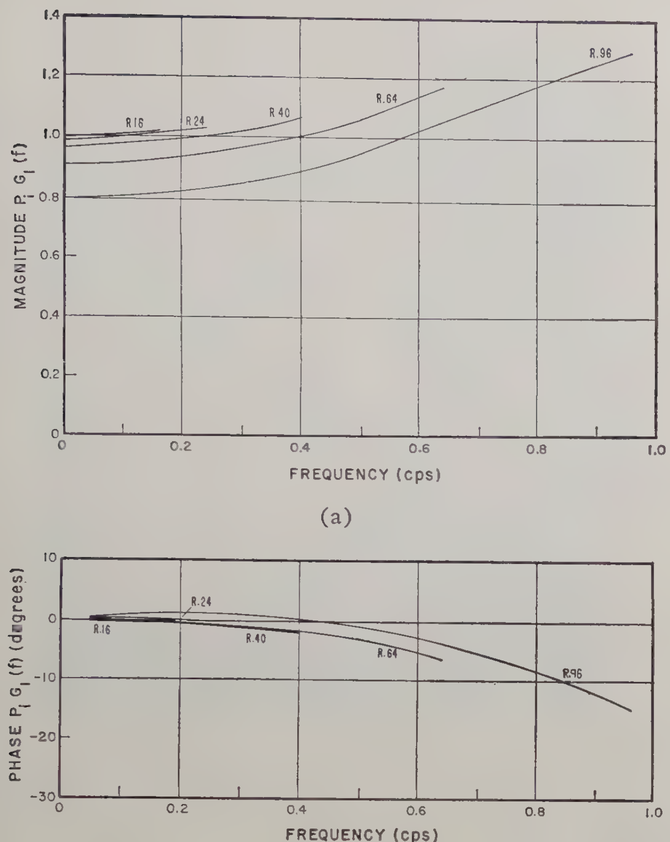
where b_0 is the displacement coefficient and b_1 the velocity coefficient. The coefficients b_0 and b_1 are found analytically to give minimum mean-square error between system input and output under the constraints imposed by $G_1(f)$.

2) Comparison of Pursuit Model and Measured Closed-Loop Transfer Functions: The parameters of the optimum $P_i(f)$, (17), were computed for the Rectangular Spectra and for the RC Filtered Spectra. $G_1(f)$ was approximated by a pure delay of 0.194 second in these calculations.

Comparison of the computed transfer functions $P_iG_1(f)$ shown in Fig. 13 with the corresponding measured functions of Fig. 14 shows that, for the Rectangular Inputs, the agreement between measured and computed is fair for low-bandwidth but deteriorates for high-bandwidth inputs. For bandwidths of 0.96 cps and greater, the agreement is poor. This was to be expected, since the calculations for $P_i(f)$ were based on the assumptions that the tracking task was simple and that the error and noise were small. Good agreement was obtained between measured and calculated transfer functions for the RC Filtered Inputs.

Almost always, the phase of the model $P_iG_1(f)$ is con-

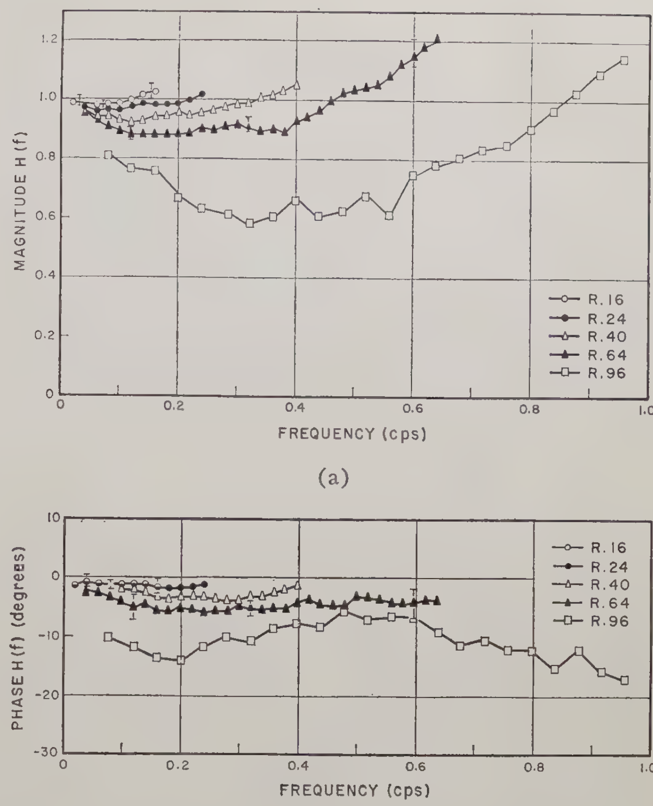
[20] W. E. Hick and J. A. V. Bates, "The Human Operator of Control Mechanisms," Permanent Records of Res. and Dev. No. 17.204, British Ministry of Supply; May, 1950.



(a)

(b)

Fig. 13—Calculated magnitude (a) and phase (b) of $P_iG_1(f)$ of pursuit model for Rectangular Inputs R.16 to R.96. The labels on the curves indicate the input bandwidth (see Section III).



(a)

(b)

Fig. 14—Experiment III (bandwidth) pursuit—mean closed-loop characteristics for Inputs R.16 through R.96. The legend on the curves refers to input bandwidth (see Section III). Vertical bars indicate standard deviations.

siderably less than the phase of measured closed-loop transfer functions, $H(f)$, indicating that the prediction obtained by the human operator is less than optimum. Probably his estimate of input displacement and velocity is degraded by noise or errors introduced by the visual process. The best agreement between measured and computed results was obtained with Input F1, broad-band noise passed through one RC filter, for which estimation of input displacement but not velocity is required for optimum prediction. These results are presented in Elkind.[6]

3) Model for Pursuit Noise: Expression (15) derived for the magnitude of the noise spectrum in the compensatory system, can be applied to the pursuit system. The assumptions made in the derivation also apply to the pursuit system. In Fig. 15 are plotted measured values for the magnitude of the noise spectrum, c_n^2, against computed values, c_p^2.

Except for band-pass inputs B9 and B10 and rectangular input R.16, the plotted points are well distributed about the line of unity slope. The correlation of all points is 0.843, which indicates that (15) accounts for most of the variance in the measured magnitudes of the noise spectra. The large discrepancies observed for band-pass inputs B9 and B10 probably are results of phase- or frequency-modulation noise produced by the

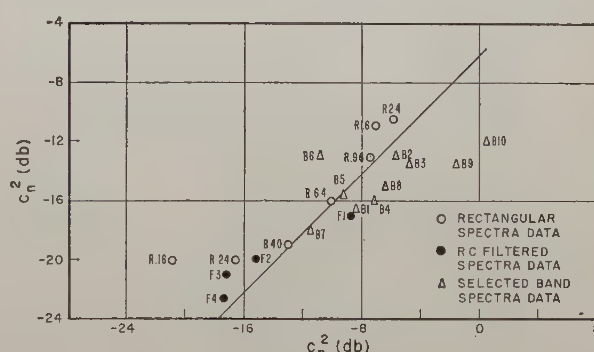


Fig. 15—c_n^2, measured magnitude of noise power-density spectrum vs c_p^2, computed magnitude, for all pursuit results.

human operator when tracking these inputs. The fact that a single relation for the noise approximates reasonably well both pursuit and compensatory systems provides further confirmation that the expression (15) is a valid one.

V. Conclusions

The analytic models, particularly those derived for the compensatory system, constitute the most important contribution of this study. The models together with the graphical results make possible prediction of performance of simple manual control systems over a wide range of input signals. Moreover, the models and

results make possible partial prediction of some characteristics of complex systems. For example, because the models apply to very simple systems, they probably represent an upper bound on human operator performance that cannot be exceeded in complex systems. Also we would expect that the relations between input signal parameters and human operator characteristics in complex systems would have many of the same tendencies as were observed in simple systems.

The following is a summary of the mathematical models for compensatory and pursuit systems. The measured open-loop transfer functions of the compensatory system $G(f)$ (Fig. 5) can be approximated by

$$G_a(f) = \frac{K_e^{-2\pi j f \alpha}}{\left(\frac{jf}{f_0}+1\right)\left(\frac{jf}{f_1}+1\right)}$$

where

$$K = \frac{0.39}{\bar{f}\sigma_f} \quad \text{or} \quad \frac{3.8}{\sigma_f e^{\bar{f}/0.38}}$$

$$f_0 = \frac{1.5}{K} \text{ cps}$$

$$\alpha \doteq 0.13 \text{ second}$$

f_1 is about 10 f_0 and is chosen so that $G_a(f)$ is stable in the closed loop.

The closed-loop (Fig. 4) noise power-density spectra for both compensatory and pursuit systems can be approximated by the quadratic

$$\frac{\Phi_{nn}(f)}{\int_0^\infty \Phi_{ii} df} = \frac{c_n^2}{\left[\left(\frac{jf}{f_n}\right)^2 + 2\zeta\frac{jf}{f_n} + 1\right)^2}$$

where

$$c_n^2 \propto \frac{H_a^2 \int_0^\infty f^2 \Phi_{ii} df}{f_n \int_0^\infty \Phi_{ii} df}.$$

For the compensatory system, f_n is approximately equal to the phase cross-over frequency of $G_a(f)$, and ζ is between 0.8 and 1.0.

The elements of the open-loop block diagram for the pursuit system (Fig. 6) are postulated to have the following forms. The inherent characteristics

$$G_1(f) = \frac{e^{-2\pi j f \alpha}}{\left(\frac{jf}{f_m}\right)^2 + 2\zeta\frac{jf}{f_m} + 1}$$

where

$$\alpha \approx 0.13 \text{ second}$$

$$f_m \approx 4.0 \text{ cps}$$

$$\zeta \approx 0.8.$$

The predictor

$$P_i(f) = b_0 + b_1(2\pi j f)$$

where b_0 and b_1 are computed to give minimum mean-square error. The feedback $G_2(f)$ has the same form as $G_a(f)$ for compensatory system except that the delay is omitted.

The models for the compensatory systems are simple and yet provide a good description of human operator characteristics. The parameters of the models are few in number and are simply related to certain other parameters which describe predictability and location in frequency of the power spectra of the input signals. The models for the pursuit system, although only semi-quantitative, demonstrate the manner in which the operator attempts to optimize his characteristics. The well-defined behavior of the compensatory models suggests that the human operator's characteristics in the pursuit system, and even in more complicated manual control systems, also have a well-defined structure. Since human operator characteristics are complicated and highly nonlinear, it is remarkable that quasi-linear models of simple form describe them so well and that relatively simple linear and quasi-linear design and analysis techniques can be applied to manual control systems.

Appendix I

Apparatus and Procedure

The apparatus used is shown in Fig. 16. The subject was seated directly in front of the upper cathode-ray tube on which was presented the target dot and follower circle. All movement was along the horizontal.

The control was a small pencil-like stylus which the subject moved on the screen of the lower oscilloscope located to the right of the subject. A voltage proportional to the position of the stylus on the lower screen was generated by an electronic circuit[21] connected to the stylus. The sensitivities of the horizontal amplifiers of both oscilloscopes were adjusted so that a movement of the stylus produced movement of the circle equal in magnitude and in the same direction. Because the stylus was light (35 grams) and frictionless, its dynamics can be approximated by a constant K_c (see Fig. 1 and Fig. 2), in this case having a value of unity. The stylus had the familiar feel of a pencil and was simple to operate. The apparatus was designed so that the tracking task would be as simple and as natural as possible.

One group of three subjects, members of the staff of M.I.T., was used in the two experiments. Before data were recorded, the subjects were trained to high proficiency. Pursuit and compensatory tests were performed separately rather than in alternation to minimize transfer effects from one to the other.

Each set of input signals was recorded on magnetic

[21] This device called a pip-trapper was developed by B. Watters at the Mass. Inst. Tech. Acoustics Lab.[6]

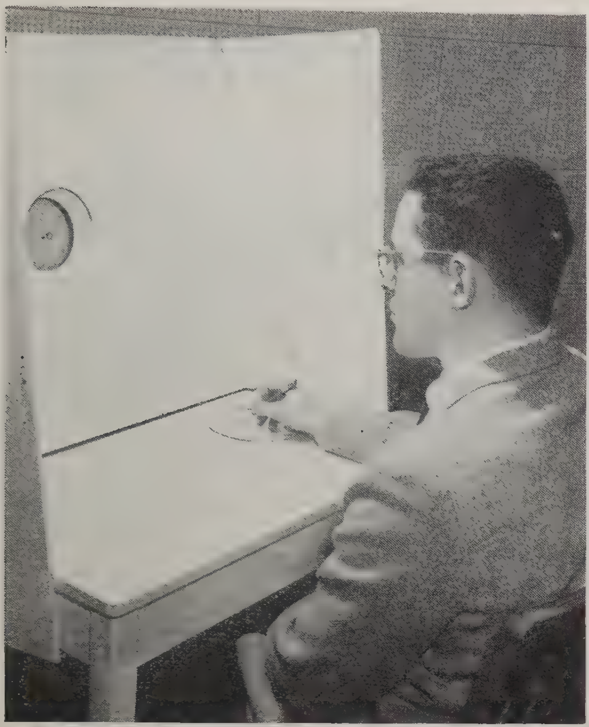


Fig. 16—Tracking apparatus.

tape. The subjects tracked each set twice. The duration of each tracking run was five minutes. The first minute was practice, for the subject to adjust to the characteristics of the signal, and the last four minutes constituted the scoring run.

The subjects were instructed to keep the center of the follower as close as possible to the target at all times. For low-bandwidth inputs, this instruction seemed unambiguous. However, for certain high-bandwidth inputs with the pursuit system, two different tactics were employed by the subjects. The first was to try to reproduce the input waveform as closely as possible; the second was to track only the low-frequency components of the input in an attempt to reduce tracking error. All subjects employed the first tactic for all input signals. In addition, two subjects tracked Inputs R2.4 and R4.0, and one subject tracked Input F1 using the second tactic.[22]

Acknowledgment

The authors are indebted to Dr. J. C. R. Licklider, who, in supervising this work, provided most valuable inspiration and encouragement and contributed many important suggestions. The guidance provided by Prof. G. C. Newton and Dr. R. C. Booton and their excellent suggestions and criticisms are greatly appreciated. The authors are grateful to R. T. Mitchell and E. Cramer for serving as subjects, J. W. Doyle for assisting in the design and construction of the electronic equipment, P. Zartarian for assisting in the computation of the results, and E. P. Brandies for performing some of the early experiments of the study.

[22] See Section III for discussion of the input signals.

Transportation Lag—An Annotated Bibliography*

ROBERT WEISS†

Summary—**This bibliography is an attempt to survey the writings, in various fields of study, which deal with functions with retarded argument. This problem is characterized by a response to a stimulus which is identical to a normal response except that it is delayed in time. Some situations in which this transportation lag occurs include process control (distance-velocity lag), control of thermal systems (including control of nuclear reactors), rocket motor combustion (ignition and combustion lags), traveling waves, magnetic amplifiers, human link in control systems (reaction time), high-speed aerodynamic control, and economic systems (period of gestation or production lag). A bibliography is presented which lists and abstracts a number of the references dealing with this problem. Relevant references in two major categories have been omitted in this bibliography. Foreign language works and pure mathematical treatments have not been listed. A comprehensive list of these references may be found in an excellent bibliography on the subject.[1]**

The format of the reference listings is an adaptation of standard bibliographical format convenient to the type of material presented. References are lettered according to the author's surname, rather than numbered, to facilitate future additions without breaking the continuity of the reference notation.

Bibliography

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Graphical solution of equation: $z_n^2I+z_nR+Sz_n \exp(-z_nT)+K=0$ in order to imply that steady-state oscillatory condition is unlikely, and may be neglected in deriving stability criteria. Discussion of Minorsky's mechanical model [Mi 2, 3] and analysis by means of vector representation. Analysis of basic control system with retarded position, rate and acceleration feedback using Cauchy Index Theorem, and derivation of stability criteria in terms of system parameters I, R, S, and K. Ranges of time lag for stability are found.

[An 2] ———, ———, vol. 18, p. 114; 1951.

Discussion of [Gu 2] reviewing various ways of analyzing system with time lag. Introduces Cauchy-Rouche theorem and investigates sufficient condition for stability of the system.

[An 3] ——— and J. A. Krumhansl, *Q. Appl. Math.*, vol. 6, p. 337; 1948. "A general stability criterion for linear oscillating systems with constant time lag."

Summary of [An 1].

[Ba 1] Barker, R. H., *Automatic and Manual Control*, ed., A. Tustin, Academic Press, New York, N. Y., pp. 461–463; 1952.

Discussion of [Ol 1]. Describes Z-transform methods of analyzing same system, and instrumentation of control feedback to obtain required transfer function for optimized control.

[Ba 2] Bateman, H., *Bull. Amer. Math. Soc.*, vol. 51, pp. 601–646; 1945. See esp. pp. 618–626, "The control of an elastic fluid."

Review of the accomplishments of mathematics in relation to physical problems, specifically those pertaining to control. Consideration of the work done on the "transcendental problem" which appears in time lag, feedback systems, and integral equations. Time-lag problem is attacked by three methods—the Taylor theorem, differential-difference theory, and integral equations.

* Revised manuscript received by the PGAC, February 5, 1959.
† Lockheed Missiles and Space Div., Sunnyvale, Calif.
[1] N. H. Choksy, "Analytic Determination of the Stability of Servomechanisms and other Automatic Control Systems Having Transcendental Characteristic Equations," Ph.D. dissertation, University of Wisconsin, Madison; 1955.

[Be 1] Beckhardt, A. R., "A Theoretical Investigation of the Effect on the Lateral Oscillations of an Airplane of an Automatic Control Sensitive to Yawing Accelerations," NACA TN 2006; January, 1950.

Analysis of the effect of an autopilot on the lateral, or Dutch Roll, mode of operation of an airplane. Results of the analysis by means of the exact time-delay operator $e^{-\tau s}$ as well as a three-term series approximation are presented in graphical form. Results are compared on the basis of period and damping of the high- and low-frequency lateral modes for varying time delay and gain. It is also shown that the approximation of a time delay by the first few terms of its series expansion may lead to erroneous results. Sample calculations for the unlagged and the lagged system are given.

[Be 2] Bellman, R., "A Survey of the Mathematical Theory of Time-Lag, Retarded Control and Hereditary Processes," Project Rand Rept. No. R-256, Rand Corp., 107 pp.; 1954.

An excellent bibliography on the mathematics of differential-difference, or hystero-differential equations. The chapters include: 1) Laplace equation; 2) Linear differential-difference equation of constant coefficients; 3) Renewal equation; 4) Asymptotic behavior of the solution of the renewal equation; 5) Systems of renewal equations; 6) Zeros of the exponential polynomials; 7) Stability of solutions of differential-difference equations; 8) Control problems; 9) Economic models; 10) Bibliography.

[Be 3] ——— and J. M. Danskin, "The Stability Theory of Differential-Difference Equations," Rand Corp. Rept. No. 381, 27 pp.; March 17, 1953.

Review of mathematical equations which depend upon past history, with emphasis on the differential-difference equations which occur in control problems. Brief survey of the stability theory of linear differential-difference equations with a presentation of the work done by Pontrjagin on the solution of transcendental equations of the form $P(z, e^z)$, with illustrative examples. Stability of solutions of nonlinear differential-difference equations is considered, reviewing the work done by Bellman, Brownell, and Wright. Excellent bibliography.

[Be 4] ———, ——— and I. Glicksberg, "A Bibliography of the Theory and Application of Differential-difference, Renewal and Related Functional Equations," Rand Corp. Memo No. RM-688, 13 pp.; 1952.

Title describes contents. Superseded by [Be 2].

[Bo 1] Boksenbom, A. S., D. Novik, and H. Heppler, "Optimum Controllers for Linear Closed-Loop Systems," NACA Tech. Note 2939; April, 1953.

Analysis of optimum controllers for general, linear, time-invariant multiloop systems. Optimizing is in terms of minimum mean-square or integral-square error for transient or statistical inputs. A turbojet engine control system is considered, with a fuel servo with dead time. Results of an experimental setup are described in which the dead time was approximated by a simple lag (break point at $T/2$) and a positive pole at $T/2$. Experimental results are in substantial agreement with analytical results.

[Bo 2] Bothwell, F. E., *Econometrica*, vol. 20, p. 269; 1952. "The method of equivalent linearization."

Discussion of quasi-linearization of Goodwin's economic model and analysis by means of a modified Nyquist criterion similar, but not identical, to Satche. Failing of Goodwin's approximation of delay by first two terms of Taylor series is discussed. Nonlinear analysis considers stability of solutions for the fundamental of the oscillation.

[Bo 3] Boxer, R. and S. Thaler, Proc. IRE, vol. 45, p. 89; 1956. "A simplified method of solving linear and nonlinear systems."

Development of a sample-data method to analyze various systems. Consideration is given to analysis of a system containing a delay line or time lag. Limitation to the method is

that the sampling interval is restricted to specific values dependent upon time lag, and output is in sampled form, but results agree with linear analysis of continuous system at sampling instants.

[Br 1] Bradner, M., *Proc. ISA*, vol. 3, p. 44; 1949. "Pneumatic transmission lag."

Experimental determination of characteristics of flow of fluid through various pipe lengths for various terminations. Experimental time delays are included on the charts.

[Br 2] Brownell, F. H., *Annals of Mathematics Studies No. 20*, ed., S. Lefschetz, Princeton University Press, Princeton, N. J., pp. 89–148; 1950. "Nonlinear delay differential equations."

Discussion of a general delay differential equation, first in view of linear theory, then by extending the theory of nonlinear integral equations. Investigation of case of nonlinear oscillations, giving asymptotic formulas for the solutions.

[Br 3] Brenner, M. M., *Proc. Natl. Simulation Council, 1957 Natl. Meeting*, "A new dead time simulator for electronic analogue computers," (Paper No. 6).

Description of magnetic-tape delay unit utilizing pulse width modulation for signal recording. Tape speed is regulated to produce variable delay. Delay ranges of 0.04 to 1.5 seconds and frequency response of dc to 30 cps are achieved to an accuracy of 0.5 per cent.

[Bu 1] Burford, T. M., *J. Appl. Phys.*, vol. 25, p. 1145; 1954. "Analysis of systems involving difference-differential equations."

Presentation of graphical method of solving characteristic exponential equation for system with time lag. Method consists of mapping "s-plane" on to "$f(s)$-plane," with s varying along constant σ and constant ω. (Similar to "polar plots" of [Fa 1]). Characteristic equation $s^2+e^{-s}(s+0.3)=0$ is used as an example for comparison with the root-locus method of [Ch 6], [Ca 1], and [Ha 2]. Exponential is plotted separately and is superimposed upon the plot of the polynomial. In this way, gain and time lag may be varied, without replotting the the entire system. Results compared favorably with those gotten with differential analyer.

[Ca 1] Callander, A., D. Hartree, and A. Porter, *(Phil.) Trans. Roy. Soc. (London)*, Series A, Col. 235, p. 415; 1935–1936. "Time-lag in a control system." (See [Ha 2] for Part II.)

One of the basic treatments of time lag in control systems. Discussion of the control equation: $n_1\theta(t)+n_2\dot{\theta}(t)+n_3\ddot{\theta}(t)+c(t+T)=0$. Three methods are investigated for solving the equation, *i.e.*: 1) Determination of "normal modes" of equation with no forcing function—accomplished by assuming a solution of the form: $Qe^{\gamma T}$, substituting in the differential-difference equation, separating real and imaginary parts, and finding limits of stability of the lower-frequency roots—system is considered stable if there are no non-negative roots; 2) Use of Heaviside operational methods and studying the system response to various inputs; 3) Numerical methods (Bush differential analyzer).

[Ch 1] Cheng, S., *Jet Propulsion*, vol. 24, pp. 27–32, 102–109; 1954. "High-frequency combustion instability in solid propellant rockets," (Pts. I and II).

Theory of unstable high-frequency oscillations is advanced by means of a simplified model. Mechanism of self-excitation is formulated, and a method of solution outlined involving substitution of an assumed solution into the equations and making simplifying assumptions. Effect of grain composition and configuration is investigated. Degree of instability is compared for the various modes of oscillations.

[Ch 2] ———, ———, vol. 24, p. 310; 1954. "Unconditional stability of low-frequency oscillations in liquid propellant rockets."

Problem of low-frequency rocket combustion stability is attacked in a slightly different manner as in [Su 1], [Cr 2], and [Ts 1]. Transfer function of the feed mechanism is investigated, and a given excitation level established for natural frequency vibrations. This given excitation is compared with the previously-used "pressure index of interaction" of the combustion process. If the latter is smaller, unconditionally stable operation results. Bi-propellant case is also considered.

[Ch 3] ———, ———, vol. 25, p. 163; 1955. "Low-frequency combustion stability of liquid propellant rocket motor with different nozzles."

Study of the effect of unstable flow in a supercritical nozzle on low-frequency stability of rocket combustion. As in previous cases ([Ch 1], [Ch 2]) simplifying assumptions are made (uniform pressure, constant flame temperature, and constant time lag), and characteristic equation is examined for neutral stability (used as stability boundary).

[Ch 4] Chien, K. L., J. A. Hrones and J. B. Reswick, *Trans. ASME*, vol. 74, p. 175; 1952. "On the automatic control of generalized passive systems."

Response of a multiple-capacity, passive system is simulated by a simple time lag and a time delay, the ratio of which becomes the characterizing parameter of the system. The time delay is approximated by 80 lag stages. Response of system is studied with variation in lag-delay ratio r, gain setting for no overshoot, and 20 per cent overshoot. System inputs were step change in desired value and step change in load. Graphs of results are plotted, and the effects of various controllers studied.

[Ch 5] Choksy, N. H., "Analytic Determination of the Stability of Servomechanism and other Automatic Control Systems Having Transcendental Characteristic Equations," Ph.D. dissertation, University of Wisconsin, 1955.

Statement and illustration of Pontryagin's [Pontryagin, L. S., "On the Zeros of Some Transcendental Functions, *Izv. Ak. Nauk S.S.S.R.*, Serii Mathematicheskii, in Russian, Moscow, USSR, vol. 6, p. 115; 1942] work on stability criterion for equations with exponential polynomials. Method utilizes properties of zero distribution in the equations, and formulates necessary and sufficient conditions for the stability and instability of this class of equations. Résumé of work done on analysis of time-lag equations. Contains a commendable bibliography, giving references to foreign language publications and mathematical treatments.

[Ch 6] Chu, Y., *Trans. AIEE*, vol. 70, pt. II, p. 291; 1951. "Feedback control system with dead-time lag or distributed lag by root-locus method."

Presentation of root-locus method of solving control systems with transporation lag. Constant phase loci are plotted for the lag and the system, and the loci of their sum plotted where the angle equals 180°. This root locus has an infinite number of branches, and for any given value of system gain, the roots of the response may be found. Callander's example ([Ca 1]) is solved, and a close agreement shown for the first few roots.

[Co 1] Cohen, G. H. and G. A. Coon, *Trans. ASME*, vol. 75, p. 827; 1953. "Theoretical consideration of retarded control."

Effect of dead-period lag on a single capacity process is considered, using various feedback configurations. Degree of stability used is a measure of the amplitude ratio of the lowest-frequency mode of oscillation. Characteristic equation $p+u+v \exp(-p)=0$ is assumed to have a solution of the form $p=(-r\pm i)$ which two equations, when solved and plotted, determine various control regions for proportional control. Other quantities such as amplitude ratio and sensitivity (gain) are indicated on these charts and design values chosen from the charts.

[Co 2] *Control Engineering* (Editors), vol. 3, no. 3, p. 6; 1956, and vol. 3, no. 5, p. 6; 1956. "The problem forum."

Problem involving an industrial situation with transportation lag presented in first issue with readers' answers in second issue. Solution involved "plant model" simulation with disturbance-response feedback.

[Co 3] Cowley, P. E. A. and A. Bremer, *Control Engineering*, vol. 4, no. 4, p. 129; April, 1957. "Dead time simulated by transport delay."

Description of a mechanical-hydraulic dead-time simulation device. Range of delays is one-half second to ten seconds, although longer delays are possible. Device uses pressure input to proportionally move pins on the periphery of a "memory wheel," which rotates at a constant speed. Output device senses position of pins at some angular displacement, and transforms this into output pressure signal.

[Cr 1] Craik, K. J. W., *Brit. J. Psych.*, vol. 38, pp. 56, 142; 1947–1948. "Theory of human operator in control systems."

Description of the analogous behavior of the human operator and servo systems. Quality of the human response is discussed, and intermittancy, damping, accuracy, and integral and differential control correlated with similar machine

quantities. Response is simulated by electrical models. Qualitative effects of reaction dead time on stability are discussed, and the nature of the "central delay" is questioned.

[Cr 2] Crocco, L., *J. Amer. Rocket Soc.*, vol. 21, p. 163; 1951, and vol. 22, p. 7; 1952. "Aspects of combustion instability in liquid propellant rocket motors" (Pts. I and II).

Relation of combustion time delay to pressure variations in rocket engine combustion chamber, and derivation of fundamental equation for the combustion chamber involving time delay. Analysis of stability of system under three operating conditions—constant rate feed, constant pressure feed, and assumed nonuniformity of time delay for different particles of propellant, using assumed exponential solution, graphical solution of characteristic equation, and investigation of stability limits. Discussion of stabilizing influence of random, nonuniform time delays. Part II extends analysis to bi-propellant systems. Prudent neglecting of various terms of the characteristic equation shows their influence upon stability of the system. Effects of flame temperature variation and nonuniform pressure distribution on the self-excited oscillations is discussed. Treatment differs from [Su 1] in that it does not assume that time lag is independent of chamber pressure.

[Cr 3] ———, and S. Cheng, *Fourth Symp. (Internatl.) on Combustion* (at M.I.T., Cambridge, Mass., September, 1952), Williams and Wilkins Co., Baltimore, Md., pp. 865–880; 1953. "High-frequency combustion instability in rockets with distributed combustion."

Extension of previous studies ([Gu 2], [Su 1], [Cr 2]) to the case of axially distributed combustion. Other assumptions remain the same. Effects of combustion index and time lag upon combustion stability for various nozzles is discussed, and charts drawn illustrating same.

[Cr 4] ———, ———, *J. Amer. Rocket Soc.*, vol. 23, p. 301; 1953. "High-frequency combustion instability in rockets with concentrated combustion."

Combustion stability problem is formulated for high-frequency oscillations and concentrated combustion at arbitrary axial position. Analysis follows and extends the previous ones ([Su 1], [Gu 2], [Cr 2], [Cr 3]). Equations are formulated and solved by judicious assumptions and neglecting of terms. Charts show the interrelation of time delay, axial position, combustion index, and vibration modes.

[Cr 5] ———, ———, "Theory of Combustion Instability in Liquid Rockets," AGARD NATO Monograph, Butterworths Scientific Publications, London, Eng., 200 pp.; 1956.

Complete treatment of previous work done in field of rocket motor stability. Contains, in organized manner, previous articles appearing in the *J. Amer. Rocket Soc.* Book covers formulation of the problem, analysis of low-frequency instability including stabilization methods, high-frequency stability, and comparisons of analytic results with experimentally-derived data. Excellent coverage of the field, with bibliography.

[Cr 6] ———, J. Grey and G. B. Matthews, *Proc. of Fifth Internatl. Symp. on Combustion*, Reinhold Publishing Corp., New York, N. Y., pp. 164–170; 1955. "Preliminary measurements of the combustion time lag in a monopropellant rocket motor."

Discussion of a preliminary attempt to measure combustion time lag.

[Cu 1] Cunningham, W. J., IRE Trans. on Electronic Computers, vol. EC-3, p. 16; December, 1954. "Time-delay networks for an analog computer."

Development of delay networks for use on analog computers. Example is worked out to illustrate method, consisting of forming ratios of polynomials by analog methods, which satisfy certain phase conditions. Set of tables is given outlining method for two-, four-, six-, and eight-root networks, each giving a certain maximum phase shift with specified departure from linear.

[Cu 2] ———, *Proc. Natl. Acad. Sci. USA*, vol. 40, p. 708; 1954. "A nonlinear differential-difference equation of growth." Also Rept. No. 5, Office of Naval Res., Nonr-433(00), Dunham Lab., Yale University, New Haven, Conn., 55 pp.; May 1, 1954.

Discussion of equation, $x'(t) = [a - bx(t-T)]x(t)$ occurring in business cycles, control of chemical reactions, and population growth. Approximate linear methods of solution are indicated, and analysis made using nonlinear, phase-plane method. Analog computer solutions are pictured for some cases considered.

[De 1] De Barber, J. P., *Trans. AIEE*, vol. 73-II, p. 372; 1955. "A magnetic tape memory for dc positional servo-mechanisms."

Description of magnetic recording technique for producing time delay for memory functions, to be used with dc positional servomechanisms. Signal modulates a Schmitt trigger whose pulses are recorded on magnetic tape. Output pulses are used to trigger an Eccles-Jordan circuit whose output is clamped and demodulated with a low-pass filter. Similar to the method used in [We 1]. Block diagrams and wave shapes given.

[Di 1] Diamantides, N. D., *Electronic Industries*, vol. 17, p. 82; 1958. "Converting recorders to rectilinear outputs."

Novel application of time delay to the conversion of curvilinear recorder outputs to rectilinear. Analog computer circuit used to achieve time delay is that described in [Ru 1].

[Du 1] Dunnegan, Jr., T. and J. D. Harnden, Jr., *Trans. AIEE*, vol. 73, p. 358; 1954. "Cyclic integrator—device for measuring frequency response of magnetic amplifiers."

Discussion of a method of investigating the time functioning of a magnetic amplifier. The transfer function is formulated, including time delay, and the effect of time delay on the firing angle of the mag-amp mentioned. (See also [St 3].)

[Dz 1] Dzung, L. S., *Automatic and Manual Control*, ed., A. Tustin, Academic Press, New York, N. Y.; pp. 13–23; 1952. "The stability criterion."

Discussion of mathematical stability criteria in terms of conformal mapping, and modification of the Nyquist Criterion. Nyquist becomes cumbersome with an expression such as:

$$F(p) = \frac{1}{\exp(p) - \alpha p} - B,$$

and a modification of the Nyquist is proposed to simplify analysis. Discussions by J. C. Gille and R. H. Tizard (pp. 40–42) are noteworthy.

[Ec 1] Eckman, D. P., *Trans. ASME*, vol. 68, p. 707; 1946. "The effect of measurement dead-time in the control of certain processes."

Description of model used to investigate relation of configuration of process, control, and dead-time. More cyclic, longer response is observed when dead-time is present.

[Ec 2] ———, ———, vol. 76, p. 109; 1954. "Phase-plane analysis—a general method of solution for two-position process control."

Application of phase-plane method of analysis to various two position (on-off) process control situations. Combinations of process lags and transportation lags considered, and examples are all illustrated. Approximation of [Zi 1] is discussed as a method for simpler systems.

[En 1] *Engineer* editorial staff, *Engineer* (London), vol. 163, p. 438; 1937.

Damping effect of time lag discussed in a mathematical note to an article on automatic control of aircraft. Typical equation of motion is cited: $x'' + Qx' + Nf'(t-h) + Pf(t-k) = 0$, and effects of various parameters investigated. Chief interest is influence of time lag on period of oscillation.

[Ev 1] Evans, D. H., IRE Trans. on Automatic Control, vol. AC-2, p. 17; 1957. "A positioning servomechanism with a finite time delay and a signal limiter."

Evaluation of response of servo system containing time delay, as well as limiting action. Analysis by complex variable theory and evaluation of residues at the closed-loop poles, located by numerical approximations.

[Fa 1] Farrington, G. H., *Fundamentals of Automatic Control*, Chapman and Hall, Ltd., London, esp. pp. 152–155 and 196–203; 1951.

Discussion of example of time delay in heat exchange of a liquid flowing through a pipe. Simple time delay is then plotted on the author's "polar chart," or conformal mapping of the "s-plane" on to an "$f(s)$-plane" (generalized Nyquist,

see [Bu 1]). Third-order system with time lag shown on the polar plot. Characteristic of the distance-velocity lag used in an assumed general plant model consisting of a time delay and exponential functions. (See [Go 2], [Co 1], and [Zi 1].)

[Fr 1] Frisch, R. and H. Holme, *Econometrica*, vol. 3, p. 225; 1935. "The characteristic solutions of a mixed difference and differential equation occurring in economic dynamics."

Consideration of the characteristic equation: $\rho = a - c \exp(-\rho\theta)$ arising in the study of business cycles, which is solved graphically for complex roots. Examination of real and imaginary roots. A first approximation is by graphical means, and iterative scheme used to refine answer. Period of oscillations and damping are discussed as a function of the empirical constants.

[Ga 1] Gates, Jr., O. B., and A. A. Schy, "A Theoretical Method of Determining the Control Gearing and Time Lag Necessary for a Specified Damping of an Aircraft Equipped with a Constant-Time-Lag Autopilot," NACA TN 2307, 38 pp.; March, 1951.

Analysis of relative stability of a linear control system with time delay as may occur in an autopilot system. Method involves substituting complex solution $a + j\omega s$ into characteristic equation and calculating conditions of gain and time constant which will yield a specific damping ratio. Results are presented on many charts and are extensively discussed.

[Go 1] Goodwin, R. M., *Econometrica*, vol. 19, p. 1; 1951. "The nonlinear accelerator and the resistance of business cycles."

Development of a nonlinear economic model and analysis by nonlinear methods. Time delay is expanded in Taylor series and all but first two terms are neglected.

[Go 2] Goodyear Aircraft Corp., "Simulation of a Process Controller," Rept. No. GER 4689, 15 pp.; March 11, 1952.

Discussion of method of simulating process control by a single lag and a time delay. Method for simulating the delay is by cascade combination of two ratios of quadratics (see [Si 2], [Th 1], [Ir 1] and [Cu 1]). Results of simulation plotted and GEDA wiring and block diagrams given.

[Go 3] Gore, M. R. and J. J. Carroll, *Jet Propulsion*, vol. 27, p. 35; 1957. "Dynamics of a variable thrust, pump-fed, bipropellant liquid rocket engine system."

Description of analog computer study of a rocket propulsion system. Transfer functions formulated and system as well as computer simulation circuits shown. Analysis makes use of Bode plots and step responses. Time lag assumed constant, but its effect on amplitude-phase plots shown. Effect of thrust level discussed. Position, velocity, and proportional-plus-velocity controllers applied to the system, and effect of the feedback gain on the system response shown.

[Gr 1] Greene, J., *Control Engrg.*, vol. 1, no. 2, p. 58; 1954. "Man as a servo component."

Equivalent transfer function for man in tracking situation is derived. System response to step and ramp inputs results in the expression

$$H(p) = \frac{2.5^{-0.4p}(p + 40)}{p(p + 15.5)}.$$

Brief mention made of instability problems introduced by time lag, but no analysis attempted.

[Gr 2] Grinya, Ya. I., and P. N. Kopai-Gora, *Automation and Remote Control*, vol. 17, p. 581; 1956. "A delay unit which utilizes decision amplifiers and capacitors."

Description of a time-delay simulator utilizing stepping-switch-capacitor memory [Iv 1] and linear interpolation for smoothing. Delay times of 0.1 second to 20 seconds with frequency response of 0.5 to 0.05 cps may be obtained with an accuracy of ± 3 per cent.

[Gu 1] Gumbel, H., "A Study of the Effect of Time Lag in Component Interaction of Systems Described by Second-Order Linear Differential Equation," Master's thesis, University of California at Los Angeles, 90 pp.; June, 1954.

Review of Taylor Expansion method of solving linear differential-difference equations with evaluation of remainder, or error term. Analytic study of effects of various order controllers on quadratic systems, with measure of "goodness" being minimum allowable damping. Analysis proceeds by the investigation of behavior of the phase angle and magnitude of the characteristic function with variation of a and b (where $s = a + jb$), and plotting results on Nyquist plots for a range of a and b.

[Gu 2] Gunder, D. F. and D. R. Friant, *J. Appl. Mech.*, vol. 17, p. 327; 1950. "Stability of flow in a rocket motor."

Study of "chugging," or low-frequency instability of rocket motors. Discussion of four methods of solving hystero-differential equations obtained in analysis: 1) step-by-step integration [Sh 1]; 2) Laplace transform; 3) assumption of exponential solution, separation into real and imaginary parts, graphical solution (neglecting higher-frequency solutions); 4) Cauchy Index and Nyqyist analysis. Methods 1) and 2) are immediately rejected. Method 3), similar to Minorsky's approach [Mi 2], is not pursued any further in the example. Method 4) is illustrated for a monopropellant and a bipropellant case.

[Ha 1] Hare Company, D.G.C., "Design and Construction of Low-Frequency Multi-Signal Correlator," Final Engrg. Rept., A. F. Cambridge Res. Center Contract No. AF 19(122)–213; 58 pp.; January 2, 1952.

Description of a magnetic tape signal correlator. Up to six signals are recorded on tape by the frequency modulation of a 1-kc carrier. Signals are then picked up on playback heads which are displaced from one another, and are demodulated, multiplied, and integrated. Record-playback accuracies of 1 per cent are claimed, and over-all correlation accuracy of 2 per cent is indicated. Signal spectrum of dc to 100 cycles and delays of 0 to 20 seconds are achieved by means of re-recording techniques.

[Ha 2] Hartree, D. R., A. Porter, A. Callender, and A. B. Stevenson, *Proc. Roy. Soc.* (*London*). Series A, vol. 161, p. 460; 1937. "Time lag in a control system—II." (See [Ca 1] for Part I.)

Further analysis of problem in [Ca 1]. Method of solution is the same, and use is made of the contours of constant value of system parameters on a graph of α vs β (where system roots are $\gamma = \alpha + j\beta$) to choose optimum position for roots. Further use is made of the differential analyzer to design an optimized system, and an actual system is discussed.

[Ha 3] Hazebroek, P. and B. L. Van Der Waerden, *Trans. ASME*, vol. 72, p. 309; 1950. "Theoretical consideration on the optimum adjustment of regulators."

The best adjustment of regulators is discussed—best being defined as that with minimum mean-square error with impulse excitation. System with dead-time lag is considered. Its transfer function, $\Phi(p) = p/[p + (ap + b) \exp(-p)]$ is graphed for imaginary p, and stability region determined for the system. (See [Mi 1]).

[He 1] Hebb, M. H., C. W. Horton, and F. B. Jones, *J. Appl. Phys.*, vol. 20, p. 616; 1949. "On the design of networks for constant time delay."

Discussion of various types of networks for constant time delay. Authors consider simple LC line, *m*-derived section, Type-B compensating network, bridged-T circuit, and capacitive shunted network, and compare them as to relative performance.

[Hi 1] Hillsley, R. H., *Control Engrg.*, vol. 3, p. 154, 159–160; September, 1956. "Analyzing control systems graphically."

Graphical method of analyzing process systems by step-by-step graphical integration through each element in system. Process with dead-time lag in feedback loop is considered as as example.

[Hu 1] Hutchison, G. E., *Ann. New York Acad. Sci.*, vol. 50, p. 221; 1948. "Circular causal systems in ecology."

A general discussion of ecological systems involving "self-correcting" paths. Growth equations are briefly touched upon and instability caused by time lag in the system is mentioned.

[Im 1] Imlay, F. H., "A Theoretical Study of Lateral Stability with an Automatic Pilot," NACA Rept. No. 693, 13 pp.; 1940.

Study of a typical control situation with system lags being approximated by a dead time, and the resulting system analyzed by expansion of the lag in a Taylor series and dropping all but the first three terms. Effect of lag in system is discussed and stability boundaries presented for a given lag.

[Ir 1] Irvine, H. L., *Natl. Simulation Conf. Proc.* (January, 1956) Dallas, Tex., pp. 25.1–25.3, "Simulation of dead time."

Description of methods now being used to simulate

dead time for analog computer studies. Explains quadratic approximation method (see [Si 2], [Go 2], [Th 1] and [Cu 1]) and expansion of exp $(-Ts)$ into continued fractions.

[Iv 1] Ivanov, V. A., *Automation and Remote Control*, vol. 17, p. 349; April, 1956. Translated from *Automatika i Telemekhanika*, same issue number, by Consultants Bureau, Inc., New York, N. Y., (1957), "A delay unit which utilizes magnetic recording."

Description of tape-recording method of producing time delays. AM system is described by block diagrams, and output response pictured. Excessive errors in this type of modulation led to use of FM system, which produces wide range of delays to an accuracy of 1.5 per cent. Principle involves recording of FM signal on an endless loop of magnetic tape, and playback of the signal through a playback head and discriminator, delays dependent upon length and speed of the loop.

[Ja 1] James, R. W. and M. H. Belz, *Econometrica*, vol. 4, p. 157; 1936. "On a mixed difference and differential equation."

Discussion of [Fr 1] and extension of analysis to wider permissible range of arbitrary constants. Graphical solution of characteristic equation is indicated, and approximate asymptotic forms derived.

[Ja 2] ——— ———, ———, vol. 6, p. 159; 1938. "The influence of distributed lags on Kalecki's theory of the trade cycle."

Investigation of the effect of a nonconstant "period of gestation," or lag, on system stability. Similar to Crocco's [Cr 2] extension of combustion instability study to nonuniform lag. Numerical examples are worked out in part to illustrate effect on Kalecki's [Ka 1] model.

[Ja 3] ——— ———, ———, vol. 6, p. 326; 1938. "The significance of the characteristic solutions of mixed difference and differential equations."

Discussion of the solution of mixed differential-difference equations occurring in Kalecki's model of the trade cycle [Ka 1]. Development and solution of these equations are discussed, and the characteristics of the solutions analyzed.

[Jo 1] Johnson, C. L., "Analog Computer Techniques, McGraw-Hill Book Co., Inc., New York, N. Y., pp. 127–132; 1956.

Restatement of Padé polynomial approximation of the Laplace shift operator and simulation with an analog computer (see [Mo 1]). Computer diagrams are shown with scale factors.

[Ka 1] Kalecki, M., *Econometrica*, vol. 3, p. 327; 1935. "A macrodynamic theory of business cycles."

Study of economic theory in order to formulate and investigate influence of economic factors on cyclical fluctuations in the economic system. Lag between order and delivery of goods causes formulation to take the form of a differential-difference equation, which is solved by neglecting of higher-frequency roots, and approximating the error thus incurred. Case of real roots is discussed, and example solved to illustrate the theory.

[Ke 1] Keppler, P. W., *Trans. ASME*, vol. 64, p. 766; 1942. Discussion of Ziegler and Nichols' "Optimum settings for automatic controls" in same issue.

Discussion uses illustration of single capacity process with distance-velocity lag to demonstrate type of system correction believed to be a good addition to original article.

[Ke 2] Kennedy, J. D., *Proc. Natl. Simulation Council, 1957 Natl. Meeting*, "Summary of dead-time simulation techniques and general applications," (Paper No. 5).

Description of time-delay simulation techniques including: electronic, capacitor storage, digital and magnetic tape methods. Limiting frequencies and delays are discussed for each method with existing hardware.

[Ki 1] Kirk, D. B., *Instruments*, vol. 23, p. 1191; 1950. "The effect of transmission distance on the stability of flow-control processes."

General discussion of effects of various types of lags on different process systems, and the interaction of these lags. Analysis proceeds by the use of Bode plots and controller charts.

[Ko 1] Kozak, W. S., WESCON CONVENTION RECORD, 1958, pt. 4, "An analogue memory."

Discussion of methods used to obtain time delays for analog work and introduction of problem posed by continuously variable delays. Development of a working analog memory with continuously variable delay utilizing capacitor storage method. Unit can delay 10 cps signals for ten seconds to a stated accuracy of 1 per cent. Application of the unit to autocorrelation measurements, simulation of transport lag in process applications, solution of convolution integrals and suggested other applications.

[La 1] Lang, G. and J. M. Ham, *Trans. AIEE*, vol. 74–II, p. 152; 1955. "Conditional feedback systems—a new approach to feedback control."

Introduction of conditional feedback concept to design of control systems. New configuration has input-output response equivalent to classical system, but disturbance-output response improved. Illustrative example of system with integration and time delay used to demonstrate advantages of new system over conventional feedback system. Use of concept in human link control systems mentioned.

[La 2] ———, ———, *Trans. ASME*, vol. 78, p. 160; 1956. Discussion of [Re 1].

Addition to, and clarification of Reswick's "plant-model controller" concept, and addition of conditional feedback arrangement to improve operation. Plant characterized by time delay is again analyzed, and results compared with Reswick's.

[La 3] Langer, R. E., *Bull. Amer. Math. Soc.*, vol. 37, p. 213; 1937. "On the zeros of exponential sums and integrals."

Discussion of some theorems which aid in the solution of transcendental equations of the form:

$$\phi(z) = \sum_{0}^{n} A_j(z) e^{c_j z}$$

(Exponential Sum); and:

$$\phi(z) = z \int \phi(t) e^{zt} dt$$

(Exponential Integral).

[Le 1] Lee, Y. C., M. R. Gore, and C. C. Ross, *J. Amer. Rocket Soc.*, vol. 23, p. 75; 1953. "Stability and control of liquid propellant rocket systems."

Extension and interpretation of [Su 1]. Analysis of a bipropellant system by means of Nyquist and Satche [Sa 3] diagrams for various values of feedback gain, time lag, and system parameters. Stability boundary plots are developed which relate system gain, time lag, frequency, and system parameters for system neutral stability. Control configurations for other types of rocket feed control are discussed and analyzed for stability conditions.

[Le 2] Leonard, A., *Trans. ASME* (Symp., on Frequency Response), vol. 76, p. 1215; 1954. "Determination of transient response from frequency response."

Graphical method for determining transient response from frequency response by a graphical Fourier-like analysis. Step function is represented as an infinite number of undamped sinusoidal oscillations, and the effect of each is found from the frequency response curves. These are graphically added to obtain the transient response. Method is used in two-control system with transportation lag.

[Le 3] Levitan, S. A., *Automation and Remote Control*, vol. 17, p. 947; 1956. "Determination of the optimum controller parameters in control systems for objects with lag."

Method of approximating the closed-loop response of a controller by a time delay and a simple lag, and derivation of the optimum controller gain for a given ratio of delay to lag (see also Ziegler and Nichols [Zi 1]). Delay is simulated by a fourth-order power expansion.

[Ma 1] Marble, F. E. and D. W. Cox. *J. Amer. Rocket Soc.*, vol. 23, p. 63; 1953. "Servo-stabilization of low-frequency oscillations in a liquid bipropellant rocket motor."

Extension of [Ts 1]. Satche [Sa 3] and Nyquist Diagrams are used, and stability plots constructed for various parameters. Feedback system is developed to stabilize unstable system by changing the shape of the Satche Diagram around the unit circle to produce an unconditionally stable system.

[Me 1] Medcalf, R. J. and H. Matthews, *Instruments and Automation*, vol. 27, p. 1642; 1954. "Solving process control problems by analog computer."

Discussion of application of "abstract analogs" to study of process control. Review of some methods currently used to simulate transportation lags in process systems, such as delay line with time scaling of computer, cascaded lags, and hydraulic lines.

[Me 2] Medvedev, S. S., *Automation and Remote Control* (in English translation), vol. 17, p. 1103; 1956. "Concerning certain laws governing the function of a human operator."

Fitting of analytical expressions to the experimental responses of a human operator to various inputs. Stability boundaries are plotted as a function of system parameters and coefficients for the theoretical expressions derived which best describe the experimental transient responses.

[Me 3] Meerov, M. V., *Automation and Remote Control*, translated from *Automatika i Tekmekhanika*, vol. 18, p. 1146; 1957. "On the synthesis of structures of multiple-looped control systems including elements with lags."

Extension of previous studies of the stability of multiloop control systems to include systems containing delays. Necessary and sufficient conditions for stability of entire multiple-loop system under infinite gain are derived and the analysis extended to include systems with nonzero initial conditions.

[Mi 1] Miesse, C. C., "Fifth Symp. (Internatl.) on Combustion." Reinhold Publishing Corp., New York, N. Y., pp. 190–195; 1955. "Oscillation of the flame front between two unlike droplets in a bipropellant liquid system."

Analysis of a possible source of combustion instability in a flame front. One formulation reduces to a hyster-differential equation which is treated (like [Mi 2]) by assuming a solution of exponential form, separation of real and imaginary terms, and analysis of the stability limits in terms of system parameters (droplet size, droplet position, mixture ratio, etc.).

[Mi 2] Minorsky, N., *J. Franklin Inst.*, vol. 232, pp. 451–487, 519–551, esp. pp. 524–536; 1941. "Control problems."

General comprehensive treatment of control and stability problems arising when retarded control is applied to a stable damped pendulum. Analysis starts with the dynamic equation: $A_2\ddot{\theta}+(A_1+a_1)\dot{\theta}+a_0\theta=0$, introduces a retarded-rate feedback, and treats the new equation by expansion into Taylor Series and neglecting higher-order terms. Hurwitz Theorem is applied and the influence of some higher-frequency oscillatory roots considered. Anticipatory control is applied to compensate for some of the unstable roots. Operational notation is applied and a purely oscillatory solution assumed for the transcendental characteristic equation, which is solved graphically for first few resonant roots. These oscillatory roots occur at frequencies higher than the system synchronous frequency, and are called parasitic oscillations.

[Mi 3] ———, *J. Appl. Mech.*, vol. 9, p. 65; 1942, and *Trans. ASME*, vol. 64 p. A65; 1942. "Self-excited oscillations in dynamical systems possessing retarded actions."

The example of the damped pendulum [Mi 2] is again used to illustrate the method of analysis of systems with transportation lag. Conditions are investigated for the system to be self-oscillatory, and those necessary for stable and unstable oscillations. Vector interpretation is made, and effects of time lag, fixed system parameters, and retarded feedback gain on self-oscillations are discussed. General procedure for analyzing this type of system is given, and conclusions drawn which extend this analysis to more general systems.

[Mi 4] ———, *J. Appl. Phys.* vol. 19, p. 332; 1948. "Self-excited mechanical oscillations."

Pendulum problem ([Mi 2], [Mi 3]) is again considered $[\ddot{\theta}(t)+p\dot{\theta}(t)+q\dot{\theta}(t-T)+w_0^2\theta(t)=0]$ in the light of linear theory showing that results do not agree with physically observed data. Nonlinear formulation of the problem $[\ddot{\theta}(t)+p\dot{\theta}(t)+q\dot{\theta}(t-T)+w_0^2\theta(t)+\omega f\{\dot{\theta}(t-T),\theta(t)\}=0\mu$ is made and a closed solution of the "limit cycle" type is found in the neighborhood of the harmonic solution. These results are compared qualitatively with oscillations of the Van der Pol type in vacuum-tube circuits.

[Mi 5] ———, *Proc. 7th Internatl. Congress of Applied Mechanics*, London (1948), vol. 4, p. 43; 1948. "Self-excited oscillations in systems containing retarded actions."

General discussion of transcendental problem, and review of previous methods of solution (see [Mi 2], [Mi 1], [Gu 2], [Ca 1]). Extension of analysis to nonlinear methods [Mi 4] yields results more consistent with physical fact.

[Mo 1] Morrill, C. D., IRE Trans. on Electronic Computers, vol. EC-3, p. 45; 1954. "A sub-audio time delay circuit."

Development of a method of approximation of time delay on an electronic differential analyzer with a sixth-order approximation of the transfer function of the delay. The Padé approximation is used for a fourth-order approximation, and a sixth-order approximation, but the former, together with an auxiliary second-order term, is shown to give better accuracy. Results of the application of a complex waveform into a four-second delay simulation are shown. Highest angular frequency is limited to $12/T$ for an accuracy of two per cent.

[My 1] Myskis, A. D., *Amer. Math. Soc. Translation No. 55*, 1951. "General Theory of Differential Equations with a Retarded Argument."

Description of the work done in the field of mathematics on different equations with retarded arguments, as well as the applications of this work in other fields. Excellent bibliography.

[Ni 1] Nixon, F. E., "Principles of Automatic Controls," Prentice-Hall, Inc., New York, N. Y., esp. pp. 341–345; 1953.

Nyquist Diagram is plotted for simple system with time-delay element in the forward path. With the aid of the "*M*-Circle" concept, a value of gain is chosen for the stability for a certain magnitude time lag. Responses with various gains are computed by numerical integration and are compared.

[No 1] Nomoto, A., *Proc. 2nd Japan Natl. Congress for Appl. Mech.* (1952), Japan Natl. Committee for Theoretical and App. Mech., Science Council of Japan, Ueno Park, Tokyo, p. 359; May, 1953. "Contribution to the root locus analysis of the feedback control system."

Review of root-locus methods and consideration of root loci of systems with distance-velocity lags and distributed lags. (See also [Tr 1] and [Ch 6].) Gain is computed at points where root-loci branches cross the imaginary axis, and it is shown that the higher-frequency roots are more stable than the lower-frequency ones.

[No 2] North, J. D., "The Human Transfer Function in Servo Systems," Ministry of Supply Rept., No. WRD6/50, Directorate of Weapons Res. (Defense), 59 pp.; November, 1950.

Analysis of tracking systems, with human as servo link, by means of linear servo theory. Terms used are carefully defined, and illustrations given. Higher-frequency roots of the hystero-differential equation are shown to have high damping, and are disregarded. Stochastic nature of the human response is discussed, and formulation of the problem involves the use of probability theory. Charts are plotted which depict the variance ratio as a function of operator parameters in various tracking configurations.

[Ol 1] Oldenberg, R. C., *Manual and Automatic Control*, ed., A. Tustin, Academic Press, New York, N. Y., pp. 435–447; 1951. "Deviation dependent step-by-step control as means to achieve optimum control for plants with large distance-velocity lag."

Analysis of sampled-data systems is continued along the lines of [Sa 2]. The deviation integral is taken as a measure of control quality (area between time axis and deviation characteristic with step input to plant). It is shown that if a sampled system is compensated optimally, it compares very favorably with a continuous system using same measure of goodness, where a transport lag exists in the original system.

[Ol 2] ——— and H. Sartorius, *The Dynamics of Automatic Control* (transl. and publ. by ASME), New York, N. Y., pp. 50–52, 59–61, 63–64, 69–70, 89–91; 1948.

Description of the causes and nature of time delay. Simulation of the response of some processes by a time delay and an exponential lag. Some actual systems are treated by the assumption of an exponentially damped response in the complex domain, solution of roots graphically, and finding of time response by evaluating the residues at the first roots (base waves). For small time lags, solution by Taylor expansion and term-by-term integration is shown. Time response of a system with time lag is optimized by forcing the first pair

of roots to be a double, negative real pair. Effects of dead-time lag in a sampled control system is discussed.

[Pa 1] Palmer, P. J., *Proc. IEE*, vol. 101–4, p. 27; 1954. "The behavior of control systems with definite time delay and random disturbance."

Solution of control system equations containing time delay and random noise terms. Solution of hystero-differential equation is by trial exponential solution, separation of real and imaginary terms, and presentation of stability boundaries in two- and three-dimensional charts. Higher-frequency roots are neglected. Equations are also expressed as finite-difference equations and stability boundaries found for these equations. Stochastic methods are used to solve stochastic hystero-differential equations with noise.

[Pa 2] Paynter, H. M. and Y. Takahashi, *Trans. ASME*, vol. 78, p. 749; 1956. "A new method of evaluating dynamic response of counterflow and parallel-flow heat exchangers."

Development of method of synthesizing a model system with a similar step response to a given system. Procedure consists of matching the coefficients of an assumed response with the desired response so as to synthesize the desired response with combination of lags, time delays (distributed and lumped), root lags, and linear functions. It is shown that this method also gives, directly, a fair approximation to the low-frequency part of the frequency response of the system.

[Pi 1] Pipes, L. A., *J. Appl. Phys.*, vol. 19, p. 617; 1948. "The analysis of retarded control systems."

Discussion of mathematical methods used to attack the problem of control systems with retarded control. General control equation is discussed. Equation with time lag is considered in operational form, and the denominator expanded so that the output takes the form of a series of steps at intervals equal to the time delay. For small values of delay, the expansion of the exponent in the characteristic equation in terms of a Taylor's expansion neglecting higher derivatives is indicated. An example is discussed and the solution of the transcendental characteristic equation by graphical methods mentioned, [Mi 3]. The Cauchy Index Theorem is suggested if information on just stability is desired.

[Re 1] Reswick, J. B., *Trans. ASME*, vol. 78, p. 153; 1956. "Disturbance-response feedback—a new control concept."

Author introduces his "disturbance response" and "plant-model controller" methods of plant control, and uses these methods to control a plant characterized by a pure time delay. Responses of this system using an optimum controller, and gain variations from optimum, are shown and discussed.

[Ro 1] Rockway, M. R., "The Effect of Variations in Control-Display Ratio and Exponential Time Delay on Tracking Performance, WADC Tech. Rept. No. 54-618, 20 pp.; December, 1954.

Results of experiments in two-dimensional compensatory tracking problem with human link, which point up relationships between exponential time delay and feedback gain. Difference between "exponential delay" and "transmission delay" (time lag) on the results of the problem is discussed.

[Ru 1] Rudolph, J. A., "GEDA Circuits to Obtain a Variable Time Delay," Goodyear Aircraft Corp. Letter AP-70659; June 15, 1955, 8 pp.

Extension of [St 5] and presentation of modification which enables change in time delay in analog computer approximation without undesirable transients in the output.

[Sa 1] Sabersky, R. H., *Jet Propulsion*, vol. 24, p. 172; 1954. "Effect of wave propagation in feed lines on low frequency rocket instability."

Treatment similar to [Su 1] with the exception that the assumption of incompressible liquid propellant in the fuel feed lines is not made. Assumption of constant time lag is retained.

[Sa 2] Sartorius, H., *Automatic and Manual Control*, Academic Press, New York, N. Y., pp. 421–447; 1951. "Deviation dependent step-by-step control systems and their stability."

Discussion of "step-by-step" control systems (sampled system with boxcar reconstruction), their equations, and a method for applying the Hurwitz criteria to the resulting difference equations. The general criteria for stability is that all roots of the characteristic equation must lie within a circle of unit radius in the complex Z-plane. A plant is discussed with a single process lag and a distance-velocity lag, and it is shown that this system is more stable then a system with two transfer lags of the same value.

[Sa 3] Satche, M., *J. Appl. Mech.*, vol. 16, p. 419; 1949. Discussion of [An 1].

A modification of the Nyquist, or Cauchy Index Theorem for stability is introduced. The characteristic equation: $p^2I+pR+Sp\exp(-pT)+K=0$ is reduced to two functions—one containing the exponential, the other, a rational expression in p. They are both plotted on a Nyquist Plot separately, and stability of the system deduced from the manner in which the vector of their difference rotates in the complex plane.

[Sc 1] Schiff, L. I. and M. Gimprich, *Trans. Soc. of Naval Arch. and Marine Engrs*, vol. 57, p. 94; 1949. "Automatic steering of ships by proportional control."

Formulation of equations for steering of ships with constant and exponential time lags. Solution by means of assumed solution and separation of real and imaginary parts and by Nyquist analysis. Comparison of methods. Numerical results presented in form of graphs and influence of various parameters and controllers discussed.

[Sc 2] Schroeder, W., "Analysis and synthesis of sampled-data and continuous control systems with pure time delays," Master's Thesis, University of California, Berkeley, September, 1956.

Development of a modified Z-transform for sampled-data systems and application of this method to sampled-data as well as continuous systems with pure time delays. Illustrative examples given. Stability criterion that all poles must lie in the unit circle in the Z-plane (see also [Sa 2]) is used for delayed systems.

[Se 1] Seymour, H., *The Electrician*, London, Eng., vol. 129, p. 63; 1942. "Time lag in automatic control."

Qualitative discussion of transportation lag and how it affects process control. Methods of correcting for it are mentioned, involving combinations of basic control elements.

[Sh 1] Sherman, S., *Q. Appl. Math.*, vol. 5, p. 92; 1947. "A note on stability calculations and time lag."

Review of previous work done in field and development of stability criteria, based upon Cauchy's Index Theorem, for the system: $f(z)=az^2+bz+Bz\exp(-z)+c$ (see [Mi 2]) for positive a, nonzero b, c and B, and unit time delay. For control ranges of values of system parameters, step-by-step integration is used, with retarded values replaced by values gotten by solving previous equation.

[Sh 2] ———, *J. Appl. Mech.*, vol. 19, p. 125; 1952.

Discussion of [Gu 2] and [An 2] with development of necessary and sufficient conditions for stability. Similar to [An 1] with encirclements of "-1" point as criterion.

[Si 1] Silberstein, L., *Phil. Mag.* vol. 29, p. 73; 1940. "On a hystero-differential equation arising in a probability problem."

Discussion of solution of equation $f'(y)=f(y-\eta)$ which arises in formulation of a probability problem. Analysis of equation, similar in form to transport lag equation, is by graphical methods, and by successive approximations.

[Si 2] Simon, H. A., *Econometrica*, vol. 20, p. 247; 1952. "On the application of servomechanism theory in the study of production control."

Application of block diagram, Laplace transform theory, and feedback analysis to economic systems. Simple system with production lag is considered. Lag is approximated by a distributed lag, and usual linear operational methods are used to analyze approximation.

[Si 3] Single, C. H., *Control Engrg.*, vol. 3, p. 113; October, 1956. "An analog for process lags."

Simulation of time delay by approximation of the transfer function $\exp(-Ts)$ by the product of the ratio of quadratics in s. (See also [Go 2], [Cu 1, 2], [Ir 1].)

[Sm 1] Smith, O. J. M. and H. F. Erdley, *Electrical Engrg.*, vol. 71, p. 362; 1952. "An electronic analog for an economic system."

Discussion of [Ka 1] and formulation of equations describing economic model in terms of electronic analog. Production lag is approximated alternately by a delay line, and by three cascaded time constants. Results of the simulation

are analyzed, and it is postulated that the introduction of a lag with a very long time constant would stabilize the system.

[Sm 2] ———, PROC. IRE, vol. 41, p. 1514; 1953. "Economic analogs."

Presentation of electronic analogs in the study of economic systems. Network and computer analogs explained, and uses described. Production and delivery delays are represented by pure time delays, but availability of such a unit is lacking. Block and circuit diagrams shown, and the economic implications of the responses of these systems discussed. Extensive bibliography.

[Sm 3] ——— and R. M. Saunders, *Trans. ASME*, vol. 70-I, p. 562; 1951. (Discussion of [St 4].)

Description of an electronic analog of Kalecki's economic model. Time delay is simulated by a delay line and a decreased time base.

[Sm 4] ———, "Feedback Control Systems," McGraw-Hill Book Co., Inc., New York, N. Y., Ch. 10; 1958.

Reviews importance of transport lag, discusses dead time and distributed lag in transmission lines and electric cables, measurements, analog models, graphical representations, transients, compensation of transport lags in control loops, predictor control, and posicast control systems.

[St 1] Sternfield, L. and O. B. Gates, Jr., "A Theoretical Analysis of the Effect of Time Lag in an Automatic Stabilization System on the Lateral Oscillatory Stability of an Airplane," NACA T.N. 2005; January, 1950.

Investigation of the effects of time lag on the lateral stability equations of an airplane. Frequency-response methods and graphical methods of solution are used. Separation of real and imaginary parts of assumed solution with graphical solution for roots used. Critical values of time lag are derived for neutral stability of the airplane and, motions of the airplane for other values derived and plotted.

[St 2] Storch, L., PROC. IRE, vol. 42, p. 1666; 1954. "Synthesis of constant-time-delay networks using Bessel polynomials."

Development of "maximally flat" time-delay networks by use of circuit synthesis methods. The network assumes a Cauer ladder form, and compares favorably in weight, and response to square waves, with the bridged-T and constant-*k* derived filters. (See also [He 1] and [Th 1].)

[St 3] Storm, H. F., *Magnetic Amplifiers*, John Wiley and Sons, Inc., New York, N. Y., pp. 137–138, 150–152, 228–231; 1955.

Discussion of transportation lag as it relates to saturable reactors (magnetic amplifiers). Development of block diagram of saturable reactor with external feedback and dead-time lag. (See also [Du 1].)

[St 4] Strotz, R. H., J. F. Calvert, and N. F. Morehouse, *Trans. AIEE*, vol. 70-I, p. 557; 1951. "Analog computing techniques applied to economics."

Exposition of the application of analog computing methods to mechanize models of economic systems. The electrical analog of the system is shown to exist and mechanization of Goodwin's model is outlined. Time delay is shown as a "black box," but no methods are suggested to obtain the function. Discussion of article ([Sm 3]) describes possible mechanization of delay.

[St 5] Stubbs, G. and C. H. Single, "Transport Delay Simulation Circuits," U. S. Atomic Energy Commission Rept. No. WAPD-T-38 and Supplement; 1954.

Analog simulation of time delay by "low-pass" and "all-pass" polynomial approximations. Various circuits are analyzed and their analog diagrams given, with scaling relationships for component values. Circuits are compared as to parameters, accuracy, and maximum phase shift. One of the all-pass circuits was constructed and responses to sine, square, and triangle waves plotted. Practical limitations of this approach, as well as advantages, are discussed. Supplement to the report discusses a modification of the simulation circuits which provides for simultaneous variation of both input signal and time delay.

[St 6] ———, *Automatic Control*, vol. 8, p. 47; April, 1958. "SADSAC: a sampled-data simulator and computer using stepping relays."

Description of a stepping-switch-type computer used for distributed parameter system simulation. One of the indicated uses for this computer is to simulate transportation lag. Capacitors are used to store signals at discrete times and the delayed signal is read from the switch by a contact sweeping at some time after the "writing" contact. Results are compared with those in [St 5]. Method of producing delay is similar to one suggested in [Iv 1].

[Su 1] Summerfield, M., *J. Amer. Rocket Soc.*, vol. 21, p. 108; 1951. "A theory of unstable combustion in liquid rocket motors."

Discussion of stability of rocket motor combustion. Equation $u''+Au'+Bu+Cu(t-T)=0$ is assumed to have a solution of the form: $u_n \exp(\lambda_n+jw_n)t$, real and imaginary parts separated, and roots found graphically. Solutions with positive λ_n are unstable, and methods are considered for modifying the parameters of the rocket fuel system for unconditional stability. Time lag smaller than the inverse of the critical frequency of the system is considered by expanding the differential-difference equation by Taylor Series and neglecting higher powers of $u(T)$. Discussion of meaning of foregoing analysis to rocket motor designer.

[Ta 1] Tallman, G. H. and O. J. M. Smith, IRE TRANS. ON AUTOMATIC CONTROL, vol. AC-4, p. 14; 1958. "Analog study of a dead beat posicast control."

Description and analysis of a servo compensation system which utilizes time delay in the forward path to control underdamped systems. Analog computer results with schematics. Analysis of errors and extension of method to more complex systems.

[Th 1] Thomson, W. E., IRE TRANS. ON ELECTRONIC COMPUTERS, vol. EC-4, p. 74; 1955.

Comparison of [Mo 1], [Cu 1], and the writer's previous work on synthesis of time-delay networks. Illustrative example is a third-order ratio of polynomials, and an operational circuit is given which will produce this approximation of a time delay. (See also [Go 2], [Ir 1] and [Si 2].)

[Ti 1] Tinbargen, J., *Econometrica*, vol. 3, pp. 241, 268–308; 1935. "Annual survey—suggestions on quantitative business cycle theory."

Review of [Ka 1], [Fr 1] and other work in formulation and investigation of economic oscillations. Discussion of effect of relative length of lag, and the various types of equations that are found in this field. A coordination of the various theories is given, and the effect of neglecting various terms investigated.

[To 1] Tomlinson, N. P., "Fundamental Circuits and Techniques used with Electric Analog Computers." (Esp. pp. 52–59, Time delay circuits.) Goodyear Aircraft Corp., Rept. No. AP-77079, 86 pp.; May 1, 1958.

Presentation of analog computer circuits for second- to sixth-order polynomial approximations of pure time delay.

[Tr 1] Truxal, J. G., "*Automatic Feedback Control System Synthesis*," McGraw-Hill Book Co., Inc., New York, N. Y., paragraphs 9.8 and 9.9; 1955.

Description of methods used to solve transcendental equations. Describes approximation of $\exp(-Ts)$ by Padé Approximants. Analysis of a continuous system $KG = 1/s(s+1)$ with transportation lag T, through Nyquist, Bode and root locus [Ch 6], demonstrating difficulties encountered in last method. Consideration of transportation lag in sampled-data systems ([Sa 2] and [Ol 1]) with analysis in Z-plane, and comparison of stability vs gain.

[Tr 2] ———, IRE TRANS. ON CIRCUIT THEORY, vol. CT-1, p. 49; 1954. "Numerical analysis for network design."

Introduction to and examples in the theory and application of the Z-transform method of analysis with illustration of use of this method in simple system with transport lag.

[Ts 1] Tsien, H. S., *J. Amer. Rocket Soc.*, vol. 22, p. 256; 1952. "Servo stabilization of combustion in rocket motors."

Treatment of Crocco's rocket engine stability problem [Cr 2]. Analysis is carried out by use of Satche Diagram [Sa 3] and Nyquist (Cauchy Index) Criteria. The intrinsic stability without fuel feed-rate control of the engine is discussed, and system dynamics for stabilized, as well as destabilized systems compared.

[Ts 2] ———, "Engineering Cybernetics," McGraw-Hill Book Co., Inc., New York, N. Y., Ch. 8; 1954.

Review of [Cr 2] and [Ts 1] with emphasis on the methods

of analyzing and synthesizing corrective networks for systems with transportation lag.

[Tu 1] Tustin, A., ed., *Automatic and Manual Control* (Proceedings of the Cranfield Conference—1951), Academic Press, New York, N. Y.; 1952, and Butterworths Scientific Publications, London; 1951.

Chapters of interest listed under individual authors.

[Vi 1] Vince, M. A., *Brit. J. Psych.*, vol. 38, p. 149; 1948. "The intermittancy of control movements and the psychological refractory period," (esp. Experiment IV—Psychological Refractory Period).

Description of tracking experiment investigating the response of the human operator to pulse function of varying duration. Effect of pulse width on accuracy of response is investigated. Frequency of oscillation as affected by delay time of human response is discussed.

[Wa 1] Walston, C. E. and C. E. Warren, "A Mathematical Analysis of the Human Operator in a Closed Loop Control System," AFPTRC Res. Bull. No. TR-54-96, 88 pp.

Development of transfer function of human in a compensatory tracking function with evaluation of constants and gains for some subjects. Stability criteria are developed using root-locus methods, and by separating real and imaginary parts of characteristic equation, solving, and graphing results to determine region of stability.

[Wa 2] Warrick, J. J. "Effect of Transmission-Type Control Lags on Tracking Accuracy," USAF, TR No. 5916, 1949; 18 pp.

Experimental study of the effects of transport delay in the accuracy of a compensatory tracking task by a human operator. Results are presented as "on target" time vs delay.

[We 1] Weiss, R., "A magnetic tape transportation lag simulator," Master's thesis, University of California at Los Angeles; 1957.

Description of low-cost transportation lag simulator used with an electronic analog computer. Delay is obtained by frequency modulation of input signal, and recording on magnetic tape. Delay is a function of apparent displacement of magnetic record and playback heads, obtained by varying the length of the tape loop. Theory of transportation lag with illustrative example. Photographs, frequency response, step responses, and computer diagrams. Extensive bibliography.

[We 2] Welford, A. T., *Brit. J. Psych.*, General Sec., vol. 43, p. 2; 1952. "The psychological refractory period and the timing of high speed performance—a review and a theory."

Discussion of the types of delay evidenced by the human response to stimuli. Survey with references.

[We 3] Wescott, J. H., *Trans. ASME* (Symp. on Frequency Response), vol. 76, pp. 1253–1259; 1954. "Synthesis of optimum feedback systems satisfying a power limitation," esp. p. 1258.

Two methods of synthesizing optimum feedback systems are presented: 1) Power limit—parameters optimized; 2) Power limit—mean-square error minimized. These criteria are applied to several systems, one including time lag, and their transient response to a velocity input shown.

[Wo 1] Wolfe, W. A., *Trans. ASME*, vol. 73, p. 413; 1951. "Controller settings for optimum control."

Controller system with time lag is optimized by minimizing deviation area of the response. The characteristic (stem) equation is analyzed to optimize values of systems parameters. (See also [Zi 1].)

[Wo 2] Wood, J., *Instruments and Automation*, vol. 30, p. 1720; 1957. "Controlling the pure delay plant."

Discussion of transport lag in a process and drawbacks of various types of controllers. Interrupted-type controller is chosen as a satisfactory device for controlling plant with delay.

[Zi 1] Ziegler, J. G. and N. B. Nichols, *Trans. ASME*, vol. 65, p. 433; 1943. "Process lags in automatic control circuits."

Methods are discussed for the determination of time lags in controlled processes. Response curves are investigated, and processes examined to formulate optimum controller settings for sensitivity and reset rate. (See also [Wo 1].) Method of analysis is approximation of process-reaction time by a dead time and a reaction rate, or by a dead time and a single lag, and by minimizing the area under the recovery curve, or response to unit impulse.

Fields of Application

Methods of Solution

Adaptive or Self-Optimizing Control Systems—A Bibliography*

PETER R. STROMER†

Summary—**Adaptive, self-adjusting, or self-optimizing servos are designed for operation in a slowly-changing environment as opposed to servos intended for a fixed environment. Optimalizer controls and similar devices which hunt for and adjust to a pre-set optimum condition are considered as adaptive servos. The references which follow are a selective sampling of the latest material on this subject taken from the open literature and technical reports. Servos which are designed to operate at some pre-set optimum based on pre-filtering of input signals evolved from Wiener's optimum filter theory, and, accordingly, references to the latter topic have also been included.**

ADAPTIVE, self-adjusting, or self-optimizing servos represent the latest advance in the realm fo feedback control. Some controversy has arisen over the terminology based on the use of the principles of adaptation as applied to this class of servos. It has been argued that feedback itself represents a form of adaptation wherein a control system monitors its own output and adjusts accordingly.

Drenick and Shahbender [13][1] borrowed the term "adaptive" from biology where it describes the ability of an organism to adjust itself to its environment. They defined adaptive servos as those designed for operation in a slowly-changing environment as opposed to servos intended for a fixed environment. Two conditions are to be fulfilled for adaptive control systems, namely:

1) The environment should be allowed to change slowly when compared with the frequencies of signal and noise and the system should be able to detect changes and vary its parameters accordingly.

2) The system should have a single input, comprising signal and possibly noise, and should be designed to derive all indications of environmental change either from that input, its own response to the input, or both.

Optimalizer controls and similar devices which hunt for and adjust to a pre-set optimum condition are considered as adaptive servos. They have been used to control fuel flow in aircraft jet engines, aircraft cruise control systems, and miscellaneous chemical engineering process control applications.

The latest and most comprehensive state-of-the-art survey on adaptive servos has been made by Aseltine, *et al.* [2], of Aeronutronic Systems, Inc. Aseltine reviews the various criteria upon which self-optimizing systems have been based and discusses the operation of each type of system. He separates adaptive servos into the following five classes:

Class I—*Passive Adaptation*
Systems which achieve adaptation without system parameter changes, but rather through design for operation over wide variations in environment.

Class II—*Input-Signal Adaptation*
Systems which adjust their parameters in accordance with input signal characteristics.

Class III—*Extremum Adaptation*
Systems which self-adjust for maximum or minimum of some system variable.

Class IV—*System-Variable Adaptation*
Systems which base self-adjustments on measurements of system variables.

Class V—*System-Characteristics Adaptation*
Systems which make self-adjustments based on measurement of transfer-characteristics.

The class distinctions are arbitrary with many obvious similarities between classes. Levin [28], for example, places all adaptive systems in three classes: 1) Input-sensing; 2) Plant-sensing, *i.e.*, over-all parameter-sensing; 3) Performance-criterion sensing.

The references which follow are a selective sampling of the latest material on this subject taken from the open literature and technical reports. Servos which are designed to operate at some pre-set optimum based on pre-filtering of input signals evolved from Wiener's optimum filter theory, and, accordingly, references to the latter topic have also been included.

In reviewing the literature, it should be noted that the Russians have done considerable work in this area, yet very little of their work has been translated to date. For those able to read Russian, the main source of information is the Russian periodical *Avtomatika i Telemekhanika*. This journal is now being translated into English by Consultants Bureau, 227 West 17th St., New York 11, N. Y., and it is expected that all articles from 1956 to date will shortly be available in English under the title of *Automation and Remote Control* (*U.S.S.R.*). In addition, a grant from the National Science Foundation to the Instrument Society of America has recently been announced to facilitate translations of *Avtomatika i Telemekhanika*.

A brief index precedes the bibliography, indicating specific topics listed. Whenever applicable, ASTIA Document (AD) or Publications Board (PB) numbers

* Revised manuscript received by the PGAC, January 8, 1959.
† General Electric Co., Schenectady, N. Y.
[1] Numbers in brackets refer to references in the bibliography.

are provided to facilitate ordering of documents from the Armed Services Technical Information Agency or the Office of Technical Services, respectively.

Acknowledgement is made to J. B. Lewis who provided source material and assistance in the preparation of this bibliography.

Index

[1] Anderson, G. W., *et al.* "A self-adjusting system for optimum dynamic performance," 1958 IRE National Convention Record, pt. 4, pp. 182–190.

Technique developed which allows a system to automatically adjust its parameters for optimum dynamic performance. Applicable to all linear systems. Results of system simulated on an analog computer are presented.

[2] Aseltine, J. A., *et al.*, "Impulse-response self-optimization as compared with other criteria for adaptive systems," presented at ASME Instruments and Regulators Conf., Newark, Del.; April 2–4, 1958. Published in IRE Trans. on Automatic Control, vol. AC-6, pp. 102–108; December, 1958.

Five classes of adaptive systems defined: 1) *Passive*-systems which achieve adaptation through design for operation over wide environmental variations. 2) *Input-signal*-systems which adjust their parameters in accordance with input signal characteristics. 3) *Extremum*-systems which self-adjust for maximum or minimum of some system variable. 4) *System-variable*-systems which base self-adjustment on measurements of system variables. 5) *System-characteristics*-systems which make self-adjustments based on measurement of transfer-characteristics.

A well-written state-of-the-art survey on adaptive systems with references to literature on all classes defined above.

[3] Ashby, W. R., "Design for a Brain," John Wiley and Sons Inc., New York, N. Y.; 1954.

Author proposes ideal adaptive system. Implies a learning process in the sense that biological controls adapt to their environment by compensating for any change that tends to make them unworkable. "Homeostat" described as a device capable of random adjustment of its own parameters whenever system-variable measurements exceed certain levels.

[4] Bairnsfather, R. R., "A Self-Adjusting Control System," Master's thesis, M.I.T. Instrumentation Lab., Cambridge, Mass., Rept. No. T-012; June, 1956.

A self-adjusting control system is defined as a feedback control system which is capable of adjusting its own compensation (by means of an external computer) in accordance with some criterion, here assumed to be the minimization of the system mean-square error. The purpose of such adjustment is to provide partial compensation for variation in the parameters of the controlled component and for variations in the statistics of the input. It is also of interest to see the results of adjustment on the effect of limiting. It was found that adjustment procedures are applicable to linear systems of order greater than two. However, in the presence of limiting, not much reduction in error is obtainable by adjustment procedures.

[5] Benner, A. H., and Drenick, R., "An adaptive servo system," 1955 IRE Convention Record, pt. 4, p. 8–14.

System switches between pre-set parameters and adapts in accordance with changes in input from constant speed to acceleration and vice-versa.

[6] Booton, R. C., Jr., "An optimization theory for time-varying linear systems with nonstationary statistical inputs," Proc. IRE, vol. 40, pp. 977–981; August, 1952.

Optimum filter theory. The mean-square optimization problem is stated for time-varying systems with nonstationary statistical input functions. Correlation functions are defined for nonstationary ensembles. The mean-square error is calculated in terms of these correlation functions. The integral equation defining the optimum system is determined by minimization of the mean-square error. An extension of Wiener's theory leading to the optimum design of constant-coefficient linear systems.

[7] Burt, E. G. C., "Self-optimizing systems," paper presented at *Conf. on Automatic Control, Heidelberg, Germany*, 1956; 3 pp.

Extension of Wiener's optimum filter theory. Applicable only if the variation in the statistical properties of the signal and noise inputs is restricted to changes in root-mean-square (RMS) values, with the frequency distributions remaining constant.

[8] Campbell, Graham, "Use of an Adaptive Servo to Obtain Optimum Airplane Responses," Cornell Aeronautical Lab., Inc., Rept. No. CAL-84; February, 1957.

A linear adaptive servo with finite feedback gains can cause a system's closed-loop transfer function to approximate some prescribed optimum, even though the system has widely variable open-loop transfer functions. The adaptive servo appears to be uniquely applicable to the airplane stability and control problem. The adaptive servo will perform satisfactorily in the presence of servo lags and open-loop nonlinearities.

[9] Chang, S. S. L., "An airframe pitch linear acceleration controller," *Natl. Electronics Conf. Proc.* vol. 12, pp. 134–151; 1956.

Optimum filter theory. Predictor type of controller for optimum control of a third-order nonlinear system with saturation type of nonlinearity. For large inputs, its operation is based on principles similar to phase space analysis. For small inputs, it automatically becomes a linear feedback system with parallel compensation.

[10] Cosgriff, R. L., "Servos that use logic can optimize," *Control Engrg.*, vol. 2, pp. 133–135; September, 1955.

Introduction of the use of logical decision elements in servomechanism synthesis. System described for maximizing flame temperature by controlled airflow in a temperature control system.

[11] Cosgriff, R. L., and Emerling, R. A., "Optimizing control systems," *Applications and Industry*, No. 35, pp. 13–16; March, 1958.

Use of logic circuits in the design of optimizing systems. Author uses "optimizing" as synonymous with "optimalizing." Designed on the premise that control systems can be designed so that they analog the processes of a human operator in the manual control of a desired system.

[12] Draper, C. S., and Li, Y. T., "Principles of Optimalizing Control Systems and an Application to the Internal Combustion Engine," American Society of Mechanical Engineers, New York, N. Y.; 1951.

Original use of optimalizing principle. Control device for an internal combustion engine designed so that it searches out automatically the optimum state of operation and confines operation close to this state.

[13] Drenick, R. F., and Shahbender, R. A., "Adaptive servomechanisms," *Applications and Industry*, pp. 286–292; November, 1957.

Adaptive servos defined as feedback control systems operating in a slowly-changing environment as opposed to those intended for a fixed environment. The servos thus defined should be able to detect changes and vary their parameters accordingly. Limited to systems having a single input, comprising signal and possibly noise, and deriving all environmental change information from a) the input or b) the system's own response to the input, or both.

Authors exclude ON-OFF or proportional control servos from consideration by virtue of their definition of adaptive servos.

[14] Early, James W., and Doody, Bernard J., "Application of an Adaptive Control System to an Aircraft with a Fixed-Gain Autopilot," WADC TN-56-334, AD-97159; August, 1956.

An analog simulation of the F-100 is set up and outfitted with a simulated autopilot. It is then incorporated into an adaptive control system, which contains a second-order system having the response characteristics desired of the aircraft. The aircraft output is compared with the optimum output and the difference fed back to the autopilot. Comparison of pitch angles and of pitch angular rates and utilization of the difference signal as a gain varier are studied. In addition to these parallel arrangements, a series connection of the ideal system with the aircraft is also investigated. It is concluded that the adaptive system used in conjunction with an autopilot is feasible—with certain restrictions—in attaining constant response to a given command input, but is not helpful in responses to random aerodynamic disturbances.

[15] Eckhardt, T. D., "Techniques for Advanced Flight Control," Radio Corporation of America, Airborne Sys. Lab., Waltham, Mass., Contract AF33(616)3584, AD-143, 443; June, 1956.

Adaptive servo feasibility study included. Nonlinear aircraft control system obtains a continuous measure of its own "linear phase-margin" from the amplitude of its self-maintained hunting oscillations, and then automatically maximizes its own loop sensitivity according to this selected criterion.

[16] Farber, B., "Computer circuit finds peak automatically," *Control Engrg*, vol. 1, pp. 70–75; October, 1954.

Peak-holding analog computer circuit used in a system which automatically searches in B and adjusts for the peak value of A. (A and B may be input-output relationship or similar system variables.) For computers that employ the automatic maximum-seeking system described, the computing time cannot be faster than the operating time of a solenoid switch which introduces a delay of about 50 msec.

[17] Flugge-Lotz, I., and Taylor, C. F., "Investigation of a Nonlinear Control System," NACA TN 3826; April, 1957.

Nonlinear feedback applied to a second-order system. Performance is evaluated in terms of the average value of the magnitude of the instantaneous error for band-limited inputs. It is an adaptive system in that adjustment is made based on some system variable. Phase-plane techniques are used for studying appropriate sets of coefficients and dependence of the switching times on the deviations from the optimum.

[18] Genthe, W. K., "Optimalizing control-design of a fully automatic cruise control system for turbojet aircraft," 1957 WESCON CONVENTION RECORD, pt. 4, pp. 47–57.

Range-maximizing cruise control. Uses basic optimalizing control principles originated by Draper and Li. Variable controlled is true airspeed, which is received as a synchro input from the aircraft-control data computer and repeated as a shaft position. Fuel flow is received as a voltage and is used to excite a computing potentioneter in the specific range computer.

[19] Goodman, T. P., and Hillsley, R. H., "Continuous measurement of characteristics of systems with random inputs: a step toward self-optimizing control," ASME Paper 58-IRD-5, presented at *ASME Instruments and Regulators Conf.*, Newark, Del.; April 2–4, 1958.

Control system characterized by moments of its impulse response. Changes in system parameters are detected from changes in the moments of the system impulse response. Analog computation experiments described. Applicable to systems which are slowly time-varying having inputs where statistical properties are also slowly time-varying.

[20] Graham, D., and Lathrop, R. C., "The synthesis of 'optimum' transient response: criteria and standard forms," *Trans. AIEE*, vol. 72, pt. II, pp. 272–288; November, 1953.

Reviews mathematical criteria for optimum transient response and concludes that the integral of time-multiplied absolute-value of error criterion is superior. System of optimization applicable only to servos which have a steady-state displacement error of zero when subjected to an input step function.

[21] Groginsky, H. L., "On the design of adaptive systems," 1958 IRE NATIONAL CONVENTION RECORD, pt. 4, pp. 160–167.

Parameter-controlled adaptive mechanisms designed using the method of dynamic programming. Method enables the optimal design of the control element for continuous or sampled-data systems.

[22] Harris, B., *et al.*, "Optimum decision feedback systems," 1957 IRE NATIONAL CONVENTION RECORD, pt. 2, pp. 3–10.

Information theory. Optimum conditions of operation of a communication system described. Null-reception decision feedback involves a decoding operation with two threshold levels. Relative adjustment of the two threshold levels is made as a function of time so that communication cost is minimized for a specified performance reliability. Input consists of an ideal bandlimited video signal which is sampled at the receiver at the Nyquist rate.

[23] Kalman, R. E., "Design of a self-optimizing control system," *ASME Trans.*, vol. 80, pp. 468–478; February, 1958.

Digital computer designed which acts as a self-adjusting machine for controlling an arbitrary dynamic process. The computer is roughly the size of an average filing cabinet. Its accuracy depends mainly on the accuracy of the computation of the pulse-transfer function from measurement data.

[24] Keiser, B. E., "The linear input-controlled variable-pass network," IRE TRANS. ON INFORMATION THEORY, vol. IT-1, pp. 34–39; March, 1955.

Optimum filter theory. Types of input functions specified which are capable of being processed more suitably by a variable-pass device than by a fixed, selective device. A comparison is made of the mean-square error performance of both devices. Adaptation thus achieved via prefiltering of input signals.

[25] Kerstukos, A. J., "Teleological control—it learns by doing," presented at *21st Annual Machine Tool Electrification Forum* sponsored by Westinghouse Electric Corp.; April 24–25, 1957.

Teleological control—another name for self-adaptive control—is allowed to experiment with a process until it finds the goal as determined by a set of built-in rules. Westinghouse teleological control trademarked Automex (for automatic experimenter). (Note: Automex name has been superseded by OPCON, according to latest correspondence received by the compiler.)

[26] Kirchmayer, L. K., "An optimalizing computer controller for the electric utility industry," ASME Paper 58-IRD-2, presented at ASME Instruments and Regulators Conf., Newark, Del.; April 2–4, 1958.

Use of a differential analyzer described in simulation of power system and dispatching system to determine optimum design of an automatic dispatching system developed by General Electric for the Kansas City Power and Light Co.

[27] Lang, G., and Ham, J. M., "Conditional feedback systems—a new approach to feedback control," *AIEE Trans.*, vol. 74, pt. II, pp. 152–161; July, 1955.

Conditional feedback adapts to input and disturbance changes. Feedback acts solely to reduce the influence of disturbances and thus determines the response of the system to external loads and internal parameter variations.

[28] Levin, Morris J., "Methods for the realization of self-optimizing systems," Instrument Society of America (ISA), Paper FCS-2-58, presented at *ASME Instruments and Regulators Conf.*, Newark, Del.; April 2–4, 1958.

Author defines self-optimizing systems in three classes: 1) *input-sensing*—adaptation based on input characteristic measurements (comparable to Aseltine's *input-signal* class); 2) *plant-sensing*—adaptation via parameter measurements (comparable to Aseltine's *extremum* and *system-variable* classes); 3) *performance-criterion sensing*—adaptation via measurement of some quantity which indicates the quality of performance of the system (comparable to Aseltine's classes of *system-characteristic* and/or *extremum* systems).

[29] Lewis, J. B., "The use of nonlinear feedback to improve the transient response of a servomechanism," *Trans. AIEE*, vol. 71, pt. II, pp. 449–453; January, 1953.

Deliberate nonlinearity used to produce transient response of a servomotor superior to that of a linear system for a step input. The feedback network described gives performance which approaches optimum when applied to a torque-limited system.

[30] Li, Y. T., "Optimalizing system for process control," *Instruments*, vol. 25, pp. 72–77, 190–193, 228, 324–327, 350–352; 1952.

Forcing function or test variation imposed on a process controller results in automatic adjustment of the controller through variations in the output. An internal-combustion engine is used as an example of the controlled system whose operation is held to the optimum output by control of one or two variables.

[31] Magnin, J. P., and Burnett, J. R., "A comparison of a contactor servomechanism with an average power constrained linear servomechanism," *Natl. Electronics Conf. Proc.*, vol. 11, pp. 974–986; 1955.

Contactor second-order servomechanism optimized in the sense that torque reversal points are found which minimize the settling time of a step-function response. ON-OFF type servo with intentional nonlinearity introduced to improve performance expected from a purely linear system. An optimizing system in that it has the best possible response to step inputs for a given value of maximum available torque.

[32] Markusen, D. L., and Keeler, R. J., "A noise adaptive flight path control system," *AIEE Second Feedback Control Systems Conf.*, pp. 115–127; April, 1954.

System adapts to noise level in received signal. Criterion for adjustment is an experimentally determined optimum as a function of noise level, *i.e.*, the variation of error limit and loop gain with noise level in error signal leads to optimum setting of the flight path control.

[33] Marx, M. F., "Application of a self-adaptive system to the control of airplane normal acceleration," *Proc. of the Computers in Control Systems Conf., AIEE*, pp. 177–183; October 16–18, 1957.

Multiplier-divider arrangement applied to the pitch channel of an aircraft to obtain a zero-error *g* command system. System called the pilot-airplane-link varies its own feedback gradients according to the controlled acceleration response.

[34] Neiswander, R. A., and MacNeal, R. H., "Optimization of nonlinear control systems by means of nonlinear feedback," *Trans. AIEE*, vol. 72, pt. II, pp. 262–272; September, 1953.

Intentional nonlinearity introduced to improve automatic control performance. Nonlinear system control achieved by means of controlled reversals of a saturating forcing function. Phase-plane techniques used in optimizing a second-order system response which is preceded by a switch or relay. In the type of system described there is a small region of linear response near the null within which the system transients are allowed to decay naturally.

[35] Reswick, J. B., "Disturbance response feedback—a new control concept," *ASME Trans.*, vol. 78, pp. 153–162; January, 1956.

Somewhat analogous to Lang and Ham's conditional feedback system [27]. Plant model controller suggested for process control. Technique offers a possible basis for realizing a fully self-adjusting automatic controller.

[36] Rom, J. W., *et al.*, "Self-Optimizing Aircraft Pitch Control," M.I.T. Instrumentation Lab., Cambridge, Mass., Rept. No. T-119, AD-143,638, May, 1957; 87 pp.

Modifications on an existing pitch control system attempted to allow its use in any aircraft without requiring detailed knowledge of the aircraft flight characteristics. Pitch rate damping, pitch acceleration feedback and variable feedback sensitivity used. Root-locus analyses and analog computer simulation techniques used.

[37] Serdengecti, Sedat, "Optimalizing control in the presence of noise interference," *Jet Propulsion*, vol. 26, pp. 465–473; June, 1956.

Filtering through cross-correlation method used to detect the input signal frequency component in the output of a modified peak-holding optimalizing control system.

[38] Silva, L. M., "Predictor control optimalizes control system performance," *ASME Trans.*, vol. 77, pp. 1317–1323; 1955.

Control or forcing of the output member requires that the error or deviation and its derivatives should be reduced to zero in three steps for third-and higher-order systems. Steps consist of a single period of maximum corrective action which forces the controlled variable in the direction of decreasing error, then in reverse, and finally a force-free period at the end of which the error and its derivatives simultaneously go to zero. Near optimum transient response achieved.

[39] Taylor, Charles F., "An approach to nonlinear adaptive control," Instrument Society of America (ISA), Paper FCS1-58, presented at *ASME Instruments and Regulators Conf.*, Newark, Del.; April 2–4, 1958.

Optimum control of a nonlinear process is illustrated for a second-order instrument servo. Controller is basically a digital computer characterized by multiple mode behavior.

[40] Taylor, Charles F., "Problems of nonlinearity in adaptive or self-optimizing systems," IRE TRANS. ON AUTOMATIC CONTROL, vol. AC-5, p. 66; July, 1958.

Panel discussion from 1957 PGAC Symposium on Nonlinear Control. Adaptive control defined as a method of control aimed at obtaining optimum system performance even when there exists incomplete or inexact analytical or analog model of the process that is being controlled.

[41] Tsien, H. S., and Serdengecti, S., "Analysis of peak-holding optimalizing control," *J. Aeronautical Sciences*, vol. 22, pp. 561–570; August, 1955.

Dynamic properties of a controlled system are approximated by a first-order linear system. Design charts are constructed for determining the required input drive speed and consequent hunting loss with specified time constants of the input and out put linear groups, the hunting period, and the critical indicated difference for input drive reversal.

[42] Tucker, G. K., "An adaptive humidity control system," ASME Paper 58-IRD-1, presented at *ASME Instruments and Regulators Conf.*, Newark, Del.; April 2–4, 1958.

Process control whereby system varies its own parameters to compensate for changes in operating conditions. Not fully representative of adaptive systems because the degree of self-adjustment is not optimal.

[43] Vasu, George, "Experiments with optimalizing controls applied to rapid control of engine pressures with high amplitude noise signal," *ASME Trans.*, pp. 481–488; April, 1957.

Control varies input fuel flow to a flight propulsion system to automatically produce maximum output pressure. Fuel servo regulated by continuous test-signal causing variations in fuel flow and engine pressures.

[44] White, B., "The Quarie optimal controller," *Instruments and Automation*, vol. 29, pp. 2212–2216; November, 1956.

Claimed to be the first commercial embodiment of optimal control. Based on the slope of the process reaction, or a system response controller. Illustrated is the use of the optimal controller regulating the flow of steam used to heat a process tank. Controller varies the temperature to continuously give maximum possible product for the prevailing conditions.

[45] White, Charles F., "Feedback System Testing," NRL Rept. No. 5039, PB-131345; November 20, 1957. See also IRE TRANS. ON AUTOMATIC CONTROL, vol. AC-6, pp. 79–88; December, 1958.

Analog method of servo testing. The signal generator used employs an analog of the servo-loop transfer function. Applicable for both linear and nonlinear servo systems. Sensing method described appears suitable for control of adaptive servos. Results indicate that the sensitivity of open-loop testing is achieved in a closed-loop testing procedure.

[46] Wiener, N., "Extrapolation, Interpolation, and Smoothing of Stationary Time Series, with Engineering Applications," Technology Press, Cambridge, Mass.; 1949.

Origin of optimum filtering and prediction theory. Wiener's theory assumes the input to a system consists of a random signal plus a random noise, each of which is stationary in the sense that its statistical properties are not time-varying. Response of the system is compared with the result of a desired, time-invariant, linear operation upon the input signal, and the difference is called the "error." The mean-square value of this error is used as the criterion for optimization.

[47] Young, N. H., "An automatic control system with provision for scanning and memory," *AIEE Trans.*, vol. 72, pt. 1, pp. 392–395; September, 1953.

System adjusts a control element to produce a maximum or minimum value of a resulting parameter. The control element is scanned through its entire range of adjustment and returned to the position resulting in the desired maximum or minimum effect. Consists of controlled device, limit switches, motor drive, comparator and motor control, and memory device.

Correspondence

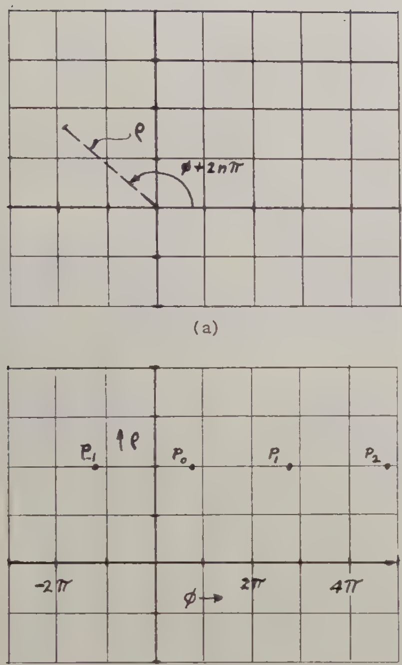


Fig. 1—(a) Polar coordinate plane, and (b) Rectangular coordinate plane.

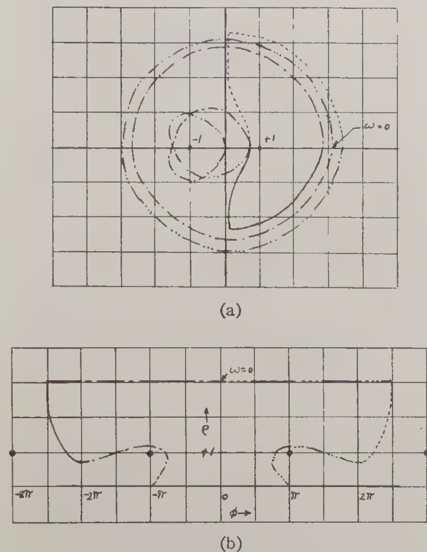


Fig. 2—Stability diagram having zero encirclements of the critical points.

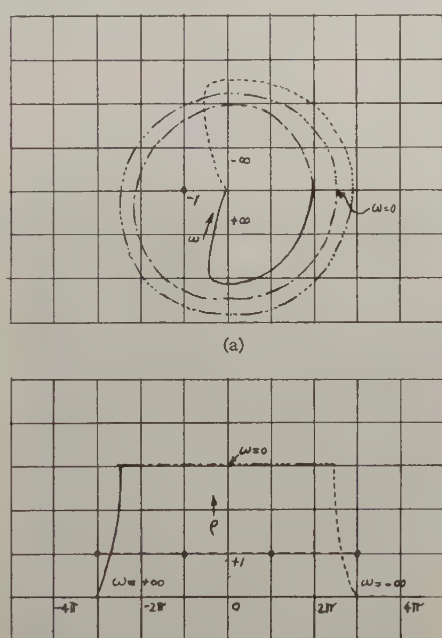


Fig. 3—Stability diagram having two encirclements of the critical points.

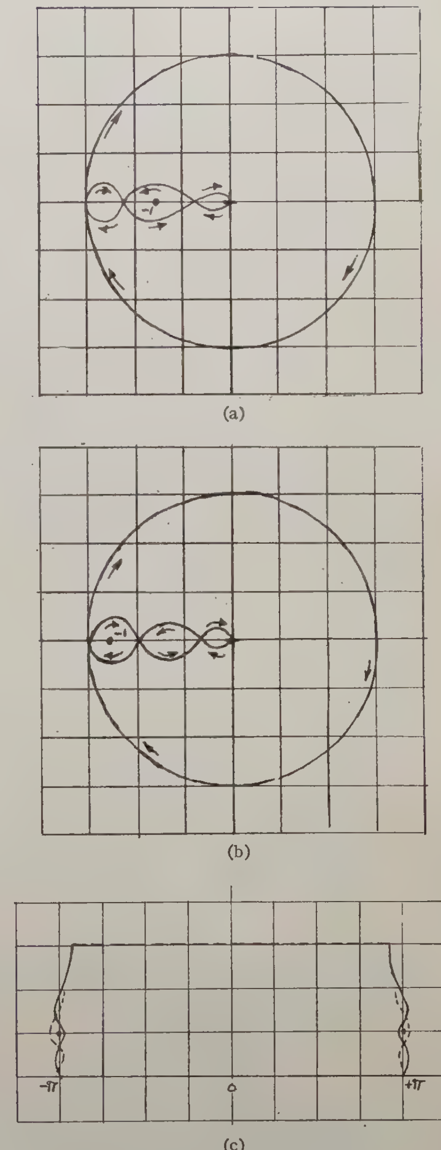


Fig. 4—(a) Conditionally stable circuit, (b) Corresponding unstable condition, and (c) Fig. 4(a) and (b) in the ρ-ϕ plane.

Stability Criteria in ρ-ϕ Rectangular Coordinates*

When a transfer function is plotted in ρ-ϕ rectangular coordinates to determine the stability of a single-loop feedback system, the stability criterion may be applied more readily than with the corresponding polar plot. Several simple examples serve to illustrate the advantages of the former coordinate system.

To determine the stability of a transfer function, $KG(j\omega)$, one frequently plots a Nyquist diagram in the ρ-ϕ polar coordinate plane, where

$$\rho(j\omega) = |KG(j\omega)|$$

and

$$\phi(j\omega) = \arg G(j\omega).$$

The multiplicity of ϕ-values for each point in this plane [see Fig. 1(a)] may result in complicated diagrams, such as those sketched in Figs. 2(a) and 3(a).

An alternative graph of $KG(j\omega)$ may be prepared on the ρ-ϕ rectangular coordinate plane [see Fig. 1(b), where several points corresponding to P in Fig. 1(a) are plotted]. The resulting diagrams are relatively simple in appearance, and the number of encirclements of the critical point

$$\lfloor \rho, \phi \rfloor = \lfloor +1, (2m+1)\pi \rfloor$$

is readily determined [see Figs. 2(b) and 3(b)].

For another example, the case of conditional stability [Fig. 4(a)] and the corresponding unstable diagram [Fig. 4(b)] given by Bode[1] are redrawn in Fig. 4(c) in the ρ-ϕ rectangular coordinate system. Again the simplicity of the latter curves is striking.

The polar coordinate graph actually is a superposition of Riemann sheets. In the ρ-ϕ rectangular coordinate plot, the m-th Riemann sheet is mapped onto the positive half-plane within the region

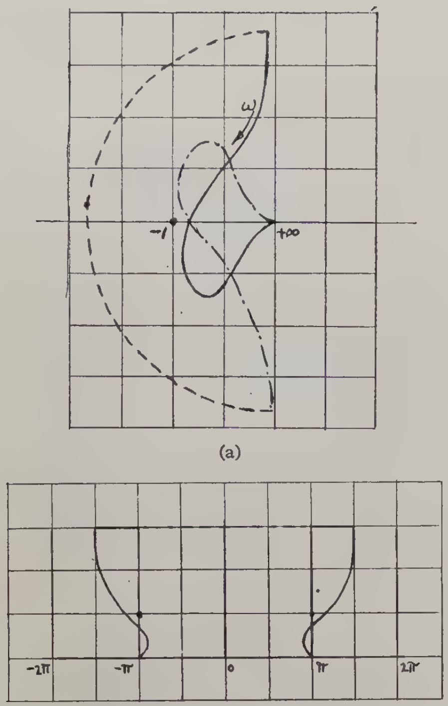
$$2m\pi < \phi < 2(m+1)\pi.$$

As a consequence, "encirclements of the critical point $(1, \pi)$" in the standard Nyquist diagram become "enclosures of the critical points $[1, (2m+1)\pi]$" in the ρ-ϕ rectangular coordinate plane.

The requirement that amplitude be an even function of frequency and phase an

* Received by the PGAC, August 4, 1958.
This work was done while the writer was associated with the Display Section, Aircraft Rad. Lab., WADC, in connection with Res. and Dev. Order No. 112-158.

[1] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. van Nostrand Co., Inc., New York, N. Y.; 1945. See especially Figs. 8.28 and 8.29, pp. 162 and 163.



(a)

(b)

Fig. 5—Examples of an incorrect stability diagram.

odd function results in even symmetry of all graphs in the ρ-ϕ rectangular system. Obviously, if a critical point is enclosed in the left half-plane, it is also enclosed in the right half-plane. We conclude, then, that the number of nulls minus the number of poles must be an even number for a function to to be physically realizable.

In Fig. 5(a) is a problem dating from the writer's school days for which there is one net encirclement of the critical point. If we plot this function in ρ-ϕ rectangular coordinates [Fig. 5(b)], we find that the curve does not close at infinity. However, another complete circle at infinity in the original diagram would result in a closed curve in Fig. 5(b); it would also yield two encirclements of the critical point, as required.

Another curve which closely resembles that of Fig. 5(a) is to be found in a well-known textbook.[2] It must be treated differently, since the actual transfer function

$$KG(s) = \frac{k}{s(s - a)}$$

is given. In this case, there unquestionably is only one encirclement of the critical point.

[2] G. S. Brown and D. P. Campbell, "Principles of Servomechanisms," John Wiley and Sons, Inc., New York, N. Y.; 1948. See especially Fig. 16(e) on p. 173 and Fig. 16(f).

While the result is mathematically correct, there is no inconsistency with the viewpoint presented in this paper, since the phase of this transfer function does not satisfy the requirement that

$$\phi(-j\omega) = -\phi(j\omega)$$

and, therefore, it cannot represent a physically-realizable network.

Rather than using ρ as the ordinate, we may employ 10 log ρ, which results in a log modulus-angle chart.[3] The critical points become [0db, $(2m+1)\pi$] and, as before, the number of critical points contained within the curve is equal to the nulls minus the poles of the transfer function that lie in the right half-plane.

With either of the rectangular plots, the number of critical points is given a plus or minus sign when $\omega = 0^+$ occurs at

$$\phi \begin{cases} <0 \\ >0 \end{cases}.$$

A pole or zero of multiplicity m must be counted m times.

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[3] *Ibid.*, Ch. 8.

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From 1949 to 1952, she was employed at the University of Utah Medical School where she was concerned with the effect of adrenal hormones on mental disorders. She joined the Lincoln Laboratory of M.I.T. at Lexington, Mass., in 1953, where she has been studying human operator characteristics and pattern recognition.

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Eliahu I. Jury (M'54–SM'57) was born on May 23, 1923, in Bagdad, Iraq. He received his undergraduate training at the Haifa Institute of Technology in Israel and was awarded the B.S. degree in electrical engineering in 1947. Subsequently, he attended Harvard University, Cambridge, Mass, receiving the M.S. degree in 1949, and Columbia University, New York, N. Y., receiving the D.Sc. degree in 1953, both in electrical engineering.

E. I. JURY

He joined the faculty of the University of California at Berkeley in 1954 as an instructor, and at present he is an associate professor of electrical engineering at the same institution. He has been very active in the field of sampled-data control systems and has served as a consultant to the Bell

Telephone Laboratories and Convair in this area. At present, he is in Europe giving a series of lectures on sampled-data systems at the University of Paris and the Eidgenossische Technische Hochschule in Zurich. He is the author of a recently published book on sampled-data control systems.

Dr. Jury is an associate member of the AIEE, a member of Sigma Xi, Eta Kappa Nu, and the Harvard Engineering Society.

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Allen S. Lange was born in Chicago, Ill., on February 17, 1921. He graduated from Lyons Township Junior College, La Grange, Ill., in 1940, with a major in English. In 1949, he received the B.S. degree in aircraft design from the University of Michigan, Ann Arbor, and in 1952, he received the M.S. degree from the same institution.

A. S. Lange

He began his industrial experience in 1940 with the Electromotive Division of the General Motors Corporation as a draftsman. Following this, he was employed by the Vought-Sikorsky Company, and served as a Production Liaison engineer. He joined the USAAF in 1943. In 1948, he was employed by the Piasecki Helicopter Company where he made field tests and design studies of control systems. In 1949, he joined the University of Michigan's Willow Run Research Center as research associate in the Rocket Motor Test Laboratory and in 1951, was transferred to the Vibration Studies Group of the Research Center as project engineer. In 1952, he joined Lear, Inc., as a research engineer, to engage in the analysis of autopilots and aircraft instruments.

Later in 1952, he joined the Instrumentation Laboratory of the Massachusetts Institute of Technology, Cambridge, Mass., as a research engineer in the applied mathematics group. In 1955, he became affiliated with the Automation Section of Raytheon Manufacturing Company as a senior engineer engaged in the design of automatic control devices for industrial applications. In 1957, he joined the staff of the Bendix Systems Division as the head of the Guidance and Control Department. In addition to the activities described, Mr. Lange has been active as a consultant in several fields, including mechanical design and packaging for the Lekas Manufacturing Company, and analog computer applications for the GPS Instrument Company.

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Ezra C. Levy (M'56) was born in Havana, Cuba, on September 22, 1924. He received the B.S. and M.S. degrees in engineering from the University of California at Los Angeles in 1949 and 1951, respectively.

He spent the five years after graduation at Douglas Aircraft Company, in Santa Monica, Calif., in the design of various guided missile control systems and autopilots, and in the next six months at Lockheed Aircraft's Missile Systems Division in Van Nuys, Calif., he designed a Supersonic target drone command guidance system and acted as interim supervisor of the Q5-flight controls group. At the time of Lockheed's scheduled relocation in Sunnyvale, Calif., he transferred to Librascope, Inc., in August, 1956, where he was engaged in the design of sampled data systems, and their applications to the control of aircraft and guided missiles. From 1954 to 1957, he acted as consultant to Dr. Travis Winsor of the Nash Cardiovascular Foundation, where he was engaged in the general problem of early detection of heart disease from electrocardiograms. In 1957, he transferred to Radioplane, a division of Northrop Aircraft, Inc., where he was engaged in the analysis and design of advanced target and reconnaissance drones, and weapon systems. He transferred to the Space Technology Laboratories, Los Angeles, Calif., in September, 1958, where he is presently engaged in the formulation of flight trajectories of space vehicles. He has been recently elected Vice-President of Varadynamics Co., an engineering consulting firm in Los Angeles, Calif.

E. C. Levy

Mr. Levy is a member of the I.A.S., A.R.S., the British Interplanetary Society, and the Institute of Navigation.

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Francis J. Mullin (S'54) was born on July 5, 1931, in Philadelphia, Pa. He received the B.S. degree from Villanova University, Villanova, Pa., in 1953, the M.S. degree from the Massachusetts Institute of Technology, Cambridge, Mass., in 1955, and the Ph.D. degree in 1959 from the University of California at Berkeley, all in electrical engineering.

F. J. Mullin

He was a teaching assistant at the Massachusetts Institute of Technology from 1953 to 1955 and was awarded an M.I.T. Summer Overseas Fellowship in 1954. In 1955, he was an associate in electrical engineering at the University of California, and from 1956 to 1958, a research assistant at the same institution. In September of 1958, he joined the faculty of the California Institute of Technology, Pasadena, where he is presently an assistant professor of electrical engineering.

Dr. Mullin is a member of the AIEE and Sigma Xi.

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Rufus Oldenburger was born on July 6, 1908, in Grand Rapids, Mich. He received the B.A. degree in Latin and Greek in 1928, and the M.S. and Ph.D. degrees in mathematics in 1930 and 1934, respectively, at the University of Chicago, Ill.

He served with the Woodward Governor Company from 1942–1957, leaving his post as director of research of the company to accept a professorship of electrical and mechanical engineering at Purdue University, Lafayette, Ind., He is now professor of mechanical engineering at Purdue, and an industrial consultant. He has held professorial chairs and other posts in mathematics from 1930–1950 at the University of Michigan, Ann Arbor, Case Institute of Technology, Cleveland, Ohio, Illinois Institute of Technology, Chicago, DePaul University, Chicago, and the Institute for Advanced Study, Princeton, N. J. He has lectured in several foreign languages and was visiting professor in universities of Mexico, France, and Japan. He introduced modern scientific techniques to the prime mover governer industry, discovered and developed various areas of linear and nonlinear control theory and applications, developed hydraulic circuit theory, and the theory of convergence of iteration methods currently used in computer and statistical fields.

R. Oldenburger

Dr. Oldenburger is President of the American Automatic Control Council, a Fellow of the AAAS, a corresponding member of the Swiss Society of Automatic Control, a member of the ASME, AIEE, Mathematical Association of America, Industrial Mathematics Society, American Mathematical Society, Society for Computing Machinery, Sigma Xi, Phi Beta Kappa, and Eta Sigma Phi.

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P. R. Stromer, for a photograph and biography, please see page 115 of the December, 1958 issue of these Transactions.

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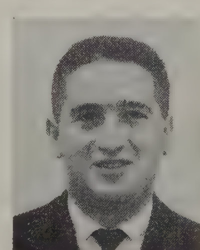
Robert Weiss (M'58) was born on February 13, 1934, in New York, N. Y. He received the B.M.E. degree in mechanical engineering in 1955 from The College of the City, New York, N.Y., and the M.S. degree in engineering from the University of California at Los Angeles in 1957.



R. Weiss

In 1955, he joined the Lockheed Missile Systems Division, Sunnyvale, Calif., and the majority of his work there has been in the Guidance and Controls Department. He has worked on autopilot development and test for the Lockheed X-7A and X-7B Ramjet Test Vehicles, and is now with the Controls Analysis Group conducting studies on missile control systems and orbit dynamics.

Mr. Weiss is a member of Tau Beta Pi, Pi Tau Sigma and Alpha Mu Epsilon.

PGAC News

1959 Dallas Control Conference

As announced in the December, 1958 issue of these Transactions, the IRE Professional Group on Automatic Control will sponsor a National Automatic Control Conference on November 4, 5, and 6, 1959, in Dallas, Texas, at the New Sheraton-Dallas Hotel. Control groups from other organizations such as the PGIE, AIEE, ASME, and ISA will participate in the activities.

Although the deadline for papers will not occur until June 1, 1959, four copies of summaries should be submitted as soon as possible to: George S. Axelby, Westinghouse Electric Corporation, Box 746, Baltimore 3, Maryland.

In order to facilitate selection of papers, the 1000- to 1500-word summary must:

1) State clearly what has been accomplished.
2) Indicate whether a) the material is primarily theoretical or experimental; b) practical applications are included; c) the paper is believed to be an original contribution or an extension of an earlier paper.
3) Include a pertinent bibliography.

Accepted papers will be published in these Transactions.

It will be noted that we are asking for long summaries because it is difficult to judge a paper from a short abstract. We also hope that the authors will heed the second and third items which are intended to give us more insight about the author, his experience, and the importance of his paper.

It should be emphasized that other organizations are involved in the Conference activities. We expect a few papers from the ISA, the PGIE, the ASME, and the AIEE. To make the Conference more of a national event, the AIEE will hold a small Control System Component Conference in the same Sheraton Hotel on November 5 and 6. The committee for this Conference will be working closely with the PGAC Conference Committee.

It was decided that the Conference should cover all phases of automatic control and to prevent restriction of subject material by implication, no theme was chosen for the Conference.

Wescon, San Francisco, 1959

The 1959 Western Electronic Show and Convention technical sessions are to be held in San Francisco from August 18–21.

This year WESCON is planning three important innovations to upgrade and enliven the technical session presentations and discussions: 1) The technical program will comprise the usual 40 daytime sessions, but with only three full-length papers in each. 2) A panel of two or three experts will be invited to comment at the conclusion of each paper. 3) The IRE WESCON CONVENTION RECORD will be available at the convention. Convention authors will be expected to submit complete manuscripts by July 1, prepared for the Record in accordance with special instructions which will be sent at the time the paper is accepted.

Adaptive Flight Control Symposium

Flight control systems probably have the greatest need for adaptive control. It is becoming more important as flight speeds and altitudes become greater and space travel becomes reality because environmental conditions and vehicle characteristics change drastically. To stress the need for adaptive control development and to illustrate the progress that has been made, a symposium on adaptive flight control systems was sponsored by the Air Research and Development Command at the Wright Air Development Center last January 13 and 14.

Presentations of adaptive control projects were made by more than a dozen organizations. Not all were confined to flight control because it was stressed that there is a serious lack of knowledge about the basic design principles of adaptive control, and progress in any area would be pertinent to the discussions.

Two organizations—the Minneapolis-Honeywell Regulator Company and the Massachusetts Institute of Technology—have successfully flight tested their respective flight control systems. Like most of the adaptive systems being studied, both of these systems maintained satisfactory aircraft control but not necessarily optimum performance during all flight conditions. Basically, the input command signals to the various aircraft control axes were passed through a second-order filter representing a satisfactory model of a desired flight control system as determined empirically. The output of this model filter was compared with the actual aircraft outputs and the resulting errors were used to direct the control surfaces in a manner which minimized the errors using normal feedback principles. However, adaptive control was used to make the feedback loops follow the filter output variations, the desired model response, with the least error at all times, even though the aircraft characteristics in the control loop varied greatly under different flight control systems.

In the M-H system, the flight control loop is 10 to 15 times faster than an ordinary aircraft control loop. It uses an OFF-ON controller, and it is essentially unstable. However, the amplitude of the oscillation is controlled to an insignificant magnitude by an automatic gain adjustment made from a measurement of the oscillation amplitude. Thus the system adapts itself to a particular operating condition, a maximum gain and bandwidth, for any flight condition or operating mode, and the adaptation takes place whenever the control-loop characteristics vary due to environmental changes, different aircraft, component aging, disturbances, or command variations. It is not necessary to measure the *causes* of the control-loop variations as it is in an ordinary compensating control system; only the *effect* of the changes is measured and used to counteract the variations. With normal control inputs, the fast control loop would not be flyable; but with the filtered input signals, the loop is not subjected to sudden commands or excessive noise. And, because it follows an input representing a satisfactory response with a minimum error, it has a satisfactory response under all conditions also. This was the basic principle investigated by the majority of participating organizations. However, the M-H mechanization was particularly simple and effective, adaptation occurred without time delay, it did not use any test signals, and it was the only one of this type extensively flight tested.

The M.I.T. system was perhaps the most sophisticated of all that were described. It has a total of seven parameters controlled from three different error criteria based on the relation between a model filter response and the actual system response in the three axes of flight control. The system was unique in that the model response was also adapted for different operating conditions of the loop. Unlike the M-H system, the control loops were not unstable in steady-state operation; and, although test signals were not used to investigate the operating condition of the control loops, it was necessary to sample the error signals periodically in order to derive the various error criteria. Internal loop gains were varied according to a logic system which essentially directed the loop variations, the adaptations, according to relations between the different criteria.

The presentations by other organizations were equally interesting and significant for the most part, but none of them had reached the stage of development of the M-H and M.I.T. systems although a few were nearly ready for actual flight evaluation. From the nature of papers given and from comments by Air Force personnel, it is evident that future aircraft control work will be of an adaptive nature.

It was emphasized that WADC is interested in determining whether these systems are the best that can be obtained. In particular, they would like to have better methods of analyzing and synthesizing these

systems. It will be desirable to know if more refined systems with learning models of complex computers will be needed for obtaining satisfactory or optimum aircraft and space flight in the future. It was suggested that industry should take more initiative in this field and that developments in this area should be reported to WADC, where the progress of adaptive control systems is being studied and recorded.

Dr. William G. Tuller Memorial Award

Each year the IRE Professional Group on Component Parts offers to a senior student or to a graduate student an opportunity to qualify for the Dr. William G. Tuller Memorial Award, which carries a stipend of $250 for the best paper submitted for publication in any IRE Proceedings, Transactions, or Symposium Record before December 31 of the preceding year. The subject may relate to operational theory, materials, construction, design, testing or application of any electronic component.

We are now considering candidates for this Award for the calendar year which closed December 31, 1958, and we would greatly appreciate your recommendation of any such student paper which appeared or was presented during that year. Will you please send your nomination directly to the attention of the writer? Your cooperation will be deeply appreciated.

Leon Podolsky
Chairman, Awards Committee
Professional Group on Component Parts
Sprague Electric Co.
North Adams, Mass.

Finite- and Infinite-State Machines

The two-week summer program at the Massachusetts Institute of Technology, Cambridge, from August 3–14, will constitute an introduction to the logical aspects of finite- and infinite-state machines. The material to be covered will range from the design and analysis of sequential machines to the theory of computability and Turing machines.

Design procedures which eliminate undesirable effects in asynchronous finite-state machines will be illustrated. The limitations and capabilities of both finite- and infinite-state machines will be characterized. Other topics will include the generation of pseudo-random sequence by linear finite-state machines and canonical forms for information-lossless circuits. The present status of and directions for future research in multidimensional logical arrays, use of unreliable components in the design of logical machines, and computation in the presense of noise will be explored.

The program will be directed by Profs. Dean Arden and David Huffman of the Department of Electrical Engineering. Other lecturers will include Profs. Edward Arthurs, S. H. Caldwell, Peter Elias, Dr. Belmont Farley, Fred Hennie, and Profs. Marvin Minsky and Hartley Rogers.

Application forms and full information may be obtained from Dr. James M. Austin, Director of the Summer Session, Room 7-103, Massachusetts Institute of Technology, Cambridge 39, Mass.

Roster of PGAC Members

Listed by IRE Regions and Sections as of April 1, 1959

The PGAC membership directory is printed annually. Note that this year membership grades are included with each name. This will help IRE sections and chapters determine which local members may be best qualified to hold positions. It may also help in stimulating member grade advancements.

Region 1

Binghamton

Angehrn, T. W.—M
Barlow, J. P.—M
Bernstein, Ralph—M
Bosman, E. H.—M
Daykin, D. R.—M
Dibb, George—M
Evans, B. O.—M
Harger, W. W.—SM
Haskins, R. J.—A
Hemstreet, H. S.—SM
Johnson, G. W.—A
Kilmer, F. G.—M
Kovalchick, Nicholas—M
Lampathakis, K. E.—M
Lohman, I. H., Jr.—M
Northrup, R. M.—M
Nuss, H. C. E.—A
Shatz, J. R.—M
Shim, I. H.—SGM
Sitterlee, L. J.—SM
Sweet, R. E.—A
Tutty, J. E.—M
Wilkinson, R. H.—M

Boston

Abate, Anthony—M
Ackerman, Sumner—M
Adams, H. E.—A
Alexander, C. S., Jr.—M
Anderegg, J. S.—A
Andreika, J. T.—M
Annis, E. W., Jr.—M
Applegate, C. E.—SM
Apsokardos, E.—A
Baker, H. W.—S
Barry, J. G.—SM
Batchelder, Laurence—F
Beaudette, C. G.—M
Beecher, A. E.—M
Benkley, F. G.—M
Bennett, H. W.—SM
Bennett,R.K.—M
Bianco, F. P.—M
Biesele, R. L., Jr.—SM
Blanchard, R. L.—SM
Blatt, Howard—A
Bliss, Z. R.—A
Bosch, F. M.—M
Boubli, E. J.—A
Bradley, C. L.—M
Brew, R. A.—M
Brown, B. D.—M
Brown, G. S.—SM
Brown, J. B.—S
Bruck, D. B.—S
Bullard, A. H., Jr.—A
Burwen, R. S.—SM
Butz, A. R.—S
Buxton, E. T., Jr.—A
Carrier, G. T.—S
Cathou, Pierre-Yves—S
Chandler, C. H.—SM
Chin, B. M.—A
Claflin, R. E., Jr.—M
Clapp, C. W.—M
Clark, G. L.—A
Clements, D. F.—M
Connelly, M. E.—M
Cook, C. G.—M
Copps, E. M., Jr.—M
Crystal, M. I.—M
Dandreta, William—VA
Demrow, R. I.—M
De Russo, P. M.—S
Dickson, A. W.—M
Dinman, S. B.—S
Dixon, L. H.—M
Dratch, J. E.—A
Dworshak, F. G.—M
Earsy, R. M.—M
Eckels, A. R.—SM
Edelstein, M. M.—S
Elkind, J. I.—M
Engel, J. S.—M
Engelberger, J. K.—A
Engman, G. E.—M
Erb, D. R.—M
Evans, R. R.—M
Fallows, E. M.—A
Farmer, J. W.—SM
Fertig, Kenneth—M
Fiddes, G. B.—M
Fitzmorris, M. J., Jr.—M
Frank, W. I.—A
Freeman, Herbert—SM
Freimer, M. L.—M
Freudberg, R. L.—A
Galagan, Steven—SM
Gansler, J. S.—M
Garber, T. A.—A
Gelb, Arthur—S
Ginns, Haskell—M
Gitelman, Ephraim—M
Glick, A. L.—M
Gocht, R. E.—S
Goldberg, David—A
Gordon, B. M.—M
Gould, L. A.—M
Goulder, M. E.—M
Grossimon, H. P.—A
Grumman, G. S.—A
Guethlen, V. J.—M
Hagan, T. G.—A
Hanks, E. C., Jr.—SGM
Hansen, J. T.—M
Hayre, H. S.—S
Healy, T. J.—S
Hersh, A. I.—M
Heuchling, T. P.—SM
Hillman, H. D.—M
Hills, F. B.—S
Hills, W. L.—M
Ho, Yu-Chi—S
Hopkins, A. L., Jr.—M
Howard, R. A.—SGM
Huibonhoa, Roger—S
Iffland, J. J.—SM
Jamgochian, Edward—SM
Johnson, E. P., Jr.—A
Jones, A. W.—SGM
Kailath, Thomas—S
Kain, R. Y.—M
Kaiser, J. F.—A
Kaye, M. G.—M
Kelleher, J. J.—S
Kennedy, R. S.—S
Kellner, Richard—M
Kerk, C. T.—A
Kipiniak, Walerian—S
Klarman, K. J.—M
Kleinrock, Leonard—S
Knight, Geoffrey, Jr.—M
Kodis, R. D.—SM
Korn, S. J.—M
Kramer, Robert—M
Krawiec, J. P.
Kremheller, W. G.—M
Kriegsman, B. A.—M
Krishnayya, J. G.—S
Krulee, R. L.—A
Lechner, R. J.—A
Leney, H. G.—M
Leonard, C. E.—M
Leonard, R. R.—M
Lincoln, A. J.—M
Lovell, R. W.—M
Lucas, E. J.—M
Luke, K. P.—S
Lynch, E. E.—SM
Mahoney, T. F.—M
Mark, R. B.—A
Markey, J. T.—A
Martin, L. H.—A
Masi, J. L.—SGM
Max, S. M.—M
McCarthy, J. F.—M
McGinn, J. F.—M
McKinney, T. D., Jr.—S
McManus, R. O.—M
McMurtrie, D. L.—SM
Meehan, D. J.—A
Mekota, J. E., Jr.—M
Melanson, F. J.—M
Mercer, W. R.—SM
Merrill, Brian—M
Miller, H. A.—SM
Minami, F. H.—S
Minsky, M. L.—M
Misek, V. A.—M
Molloy, K. H.—S
Moore, R. L.—M
Morgenstern, J. C.—A
Mori, Hideo—SM
Murano, Lodovico—M
Nagy, Ferenc, Jr.—M
Narud, J. A.—M
Naylor, T. K.—M
Neider, F. V.—S
Neidorf, Edward—A
Newton, G. C., Jr.—M
Nielsen, C. E., Jr.—M
O'Brien, D. G.—M
Oettinger, A. G.—M
Olsen, K. H.—A
Olsson, E. A.—M
Orenberg, Arthur—M
Osman, M. S.—M
Packard, R. H.—M
Palmer, P. J.—M
Parke, N. G., III—M
Pastan, H. L.—M
Pauplis, L. M.—M
Pease, W. M.—SM
Perron, R. R.—M
Perry, K. E.—A
Petteruti, A. J.—M
Phillips, D. R.—S
Pinckney, R. P.—M
Platt, H. J.—SM
Pouliot, L. J.—A
Preston, J. L.—M
Pugh, A. L., III—M
Pughe, E. W., Jr.—M
Ramamoorthy, C. V.—M
Reichard, R. W.—M
Reynolds, R. O.—S
Rigby, Sherman—M
Rittenburg, S. E.—M
Roch, M. E.—S
Rojak, F. A.—M
Rosenes, Oscar—M
Rowell, W. G.—M
Roy, A. G., Jr.—S
Sabin, E. A.—M
Sabo, J. D.—M
Sakrison, D. J.—M
Sallen, R. P.—M
Scanlon, W. C.—M
Scheidenhelm, Ralph—M
Schochet, J. R.—A
Schramm, M. W., Jr.—S
Seaver, W. H.—M
Seifert, W. W.—SM
Shansky, David—M
Sheingold, D. H.—M
Shortell, A. V., Jr.—M
Sims, B. S.—M
Sinclair, D. B.—F
Slattery, T. G.—SM
Smith, B. K.—M
Smith, D. C.—M
Smith, J. E.—F
Smith, T. B.—M
Snyder, D. C.—M
Soderstrom, R. E.—A
Solomonoff, R. J.—M
Spergel, Philip—M
Spitzer, Edward—M
Staiano, A. J.—M
Stevens, J. W.—S
Strassberg, D. D.—M
Susskind, A. K.—A
Taylor, H. P.—M
Teixeira, N. A.—M
Thaler, H. A.—S
Thayer, W. D.—M
Thoresen, J. C.—A
Tsao, C. H. K.—M
Tunnicliffe, W. W.—M
Vacca, R. H.—SM
Valpey, R. S., Jr.—A
Van Den Biggelaar, Hans—M
Vander Velde, W. E.—M
Van Wechel, R. J.—M
Vincent, G. D.—A
Ward, J. E.—M
Weinstein, I. J.—M
Wexler, H. T.—SM
Whipple, R. L.—SGM
Whitcraft, W. A., Jr.—SM
White, D. J.—M
Whittaker, H. F., Jr.—A
Wilcox, R. B.—M
Wilkie, L. E.—A
Williams, S. B.—SM
Williamson, R. J.—A
Willis, J. T.—M
Wolf, S. M.—A
Wolfe, Russell—A
Woodruff, T. E.—M
Woodward, J. H.—SM
Wozniak, E. T.—S
Yamron, Joseph—M
Young, F. M.—SM
Zames, George—S
Zantos, N. G.—M
Zieman, H. E.—M
Zisk, S. H.—A

Buffalo-Niagara

Aines, F. G.—M
Archibald, W. R.—M
Ballantine, J. H.—M
Beneke, Jack—A
Doyle, W. C.—M
Forbath, F. P.—SM
Grose, C. W.—S
Halsted, G. P.—M
Hayman, R. A.—A
Meister, L. E.—SGM
Michaelis, T. D.—M
Newton, D. J., Jr.—M
Novo, R. B.—M
Powell, F. D.—SM
Ruda, E. V.—S
Rusnak, Walter—M
Rynaski, E. G.—SGM
Savi, Lembit—M
Schneeberger, R. F.—A
Walbesser, W. J.—M

Connecticut

Angel, H. R.—M
Balducci, J. D.—S
Bohacek, P. K.—S
Bourret, C. J.—M
Briscoe, F. J.—S
Brockett, R. I.—A
Brooks, F. A., Jr.—M
Brownell, R. M.—S
Cox, D. B., Jr.—S
Curtiss, R. H.—A
Delany, E. B.—M
Dewey, C. P., Jr.—S
Dolan, W. F.—M
Ferre, M. C.—M
Flynn, T. F.—M
Garneau, M. W.—S
Georgi, E. A., Jr.—A
Gilchrist, E. S.—M
Gray, N. L.—M
Gutterman, A. Z.—M
Haas, V. B., Jr.—M
Hall, B. A.—M
Harrington, C. F., Jr.—M
Helterline, L. L., Jr.—M
Hubbard, R. G., Jr.—M
Kay, Martin—M
Lamb, J. J.—F
Lawdenslager, J. R.—A
Leuze, W. F.—A
Lindorff, D. P.—M
Lucal, H. M.—M
Lutz, C. H.—SM
Lynn, E. L., Jr.—S
Maltais, W. E.—SM
Mapes, T. J.—A
Miller, E. F.—M
Montgomery, R. H., Jr.—A
Neill, G. W.—M
Nelson, G. E.—M
Northrop, R. B.—M
Norton, C. A.—SM
Palmero, Albert—M
Penney, A. W.—M
Perkins, W. W.—SM
Pichard, L. A.—SM
Plehaty, S. L.—A
Poland, W. L.—A
Rhyins, R. W.—M
Schoenemann, P. T.—SGM
Schulman, B. L.—SGM
Selin, Ivan—SGM
Sherman, P. M.—S
Skinner, Robert—SGM
Sontheimer, C. G.—M
Stephanz, G. H.—M
Stoker, W. C.—SM
Stone, F. A.—M
Sweet, R. D.—A
Theroux, G. A.—S
Torrance, J. H.—M
Willis, P. A.—M
Zukowsky, W. S.—SGM
Zweig, Felix—M

Erie

Gray, R. B.—SM
Rochin, D. W.—A

Ithaca

Blott, E. J.—M
Chernoff, D. P.—M
Hufnagel, R. E.—S
Jackson, A. S.—M
Kalani, P. W.—S
Lind, E. R.—M
Malone, D. M.—S
Martin, A. R.—S
Mayer, H. F.—F
McLange, Thomas—S
Meserve, W. E.—SM
Phillips, T. A.—S
Scudder, H. J., III—SGM
Woodson, R. D.—SGM

Rochester

Berch, W. H.—M
Brown, G. A.—SGM
Ceranowicz, H. N.—M
Chapin, D. W.—A
Chesna, John—A
Dutcher, B. C.—M
Eisengrein, R. H.—A
Ellis, T. E., Jr.—M
Enslein, Kurt—SM
Federici, J. T.—M
Formicola, A. F.—A
Heit, J. C.—M
Johnson, R. M.—M
Merle, C. W.—SM
Morris, I. A.—M
Morse, J. E.—M
Rodatus, E. J.—A
Shalloway, A. M.—SM
Sheehan, J. F.—A
Shepard, W. H.—M
Stone, D. J., Jr.—A
Tartanian, C. N.—SGM
Thalner, R.R.—SM
Trost, H. K.—M
Trott, Marvin—M
Williams, A. S.—M

Rome-Utica

Baldridge, B. H.—SM
D'Hoostelaere, A. C.—M
Glaser, G. J.—M
Krajewski, S. J.—A
Rasmussen, C. P.—M
Rubenstein, S. E.—M
Walker, R. K.—M
Wendt, R. F.—A
Wilson, W. P.—SGM
Yoder, D. F.—A

Schenectady

Action, E. S.—M
Borck, C. A.—SM
Borner, E. F.—M
Buchhold, T. A.—SM
Chestnut, Harold—SM
Dabul, Amadeo—M
Dodson, G. C., Jr.—M
Fanuele, F. J.—S
Kirchmayer, L. K.—M
Kitsopoulos, S. C.—M
Lippitt, D. L.—A
Pester, R. F.—SM
Quinlivan, R. P.—SGM
Rothe, F. S.—M
Shuey, R. L.—SM
Stromer, P. R.—A
Stutt, C. A.—M
Swann, D. A.—S
Troutman, P. H.—M
Wright, W. H., Jr.—S

Syracuse

Ashcroft, D. L.—A
Bessette, D. U.—M
Brady, D. J.—M
Brule, J. D.—SM
Buchta, J. C.—M
Burke, T. H.—M
Clark, P. B.—M
Cottle, D. W.—A
Cover, N. W.—M
Edwards, K. A.—A
Fehlau, C. E.—M
Hannah, W. M.—A
Hatfield, J. P.—M
Heartz, R. A.—M
Huang, R. Y.—M
Johnson, G. L.—M
Jureller, J. F.—M
Kipp, K. J.—A
Lindsay, G. A.—M
Mayo, B. R.—A
McCarthy, J. J.—SGM
Neelands, L. J.—M
Russell, J. B., Jr.—SM
Schumacher, G. B.—A
Shuart, O. H.—M
Smith, G. H.—S
Somers, L. E.—M
Stabler, E. P.—A
Taylor, J. M.—M
Vandling, G. C.—M

Western Massachusetts

Ackerson, R. D.—M
Fleischer, K. M.—M
Grooms, W. G.—A
Harris, L. B.—M
Henden, Holger, Jr.—S
Lovell, B. W.—M
Luoma, R. A.—M
Mahar, T. F.—M
Patch, R. J.—M

Region 2

Long Island

Adise, H. H.—M
Agree, Irvin—M
Amato, J. A.—M
Bargeski, A. J.—M
Behn, E. R.—SM
Bibbero, R. J.—SM
Brownman, H. L.—M
Buehrle, W. E., Jr.—M
Burgess, E. G. Jr.—M
Burr, R. P.—SM
Cady, R. T.—M
Callais, R. T.—M
Cap, S. T.—M
Cardwell, R. A.—S
Carlson, W. A.—M
Caruthers, F. P.—SM
Chapman, P. W.—M
Chartoff, Philip—M
Cohen, M. J.—M
Cohen, S. L.—S
Cornetz, Walter—M
Corrado, V. M.—A
Costa, T. A.—M
Cowgill, D. E.—SM
Cramer, Sydney—M
Crepeau, P. J.—M
Croly, J. W.—A
Crosby, M. G.—F
Cullen, W. P.—M
Darden, R. R., Jr.—SM
De Gennaro, Raymond—A
Degnan, B. T.—M
De Rocher, W. L.—SM
Detwiler, S. P.—M
Di Toro, M. J.—SM
Dmytrasz, J. N.—M
Doersam, C. H., Jr.—SM
Dyer, J. N.—F
Eason, Leroy—M
Egnuss, E. M.—M
Eisele, J. G., Jr.—M
Eklund, K. E.—M
Ellis, P. H.—M
Engelson, H. R.—A
Fabricant, B. S.—M
Farber, Bernard—M
Feay, D. I.—M
Fein, Arthur—M
Fink, Albert—SM
Firth, L. G., Jr.—A
Fishbein, Milton—SM
Fonseca, A. P.—A
Franceschini, J. B.—M
Frank, F. P. E.—A
Fried, George—A
Friedman, E. D.—SM
Friedman, Ira—M
Garnett, D. W.—M
Geiss, G. R.—S
Gilbert, R. C.—SM
Gilmore, M. A.—SGM
Gister, Stanley—M
Glickman, Herbert—S
Glixon, H. P.—A
Gneses, M. I.—M
Goldberg, R. M.—M
Goodstein, Julian—M
Gordon, Fredric—M
Gordon, O. M.—SGM
Gordon, R. L.—A
Gorelick, George—M
Grabbe, Dimitry—A
Grace, M. I.—SGM
Grant, R. T.—S
Greene, G. F.—M
Gretz, R. W.—M
Gross, S. H.—SM
Grossman, A. S.—A
Guido, L. A.—A
Hall, R. L.—SM
Hansen, H. R.—A
Hansen, J. A.—SM
Harrison, Seymour—M
Hausmann, H. C.—SM
Haynes, N. M.—M
Heacock, W. J., Jr.—M
Herman, Sidney—M
Hershkowitz, Philip—M
Huber, W. J.—M
Iddings, G. E.—SM
Iernan, F. V.—M
Jaworowski, B. R.—A
Johl, M. J.—SM
Joline, E. S.—A
Jones, M. E.—SGM
Jordan, D. B.—A
Julich, Harry—M
Kanterman, Arthur—SGM
Kay, L. M.—SM
Kfoury, N. F.—S
King, L. H.—SM
Kintner, P. M.—M
Kirby, M. J.—M
Klein, R. C.—M
Kmiecik, J. E.—M
Knocklein, H. P.—A
Knox, R. W.—M
Kuhn, K. H.—M
Laspina, C. A.—M
Leary, R. W.—M
Lemanczyk, J. C.—M
Lenefsky, Selig—M
Levenstein, Harold—M
Levinson, Emanuel—M
Linden, A. J.—M
Loeffler, W. J., Jr.—M
London, F. H.—A
Marcinkowski, E. R.—M
Marcus, D. H.—M
Marston, R. S.—M
Match, M. J.—M
McPherson, D. L., III—M
Meirowitz, R. L.—M
Melman, Myron—M
Menes, Jack—S
Meyer, D. P.—M
Mooney, V. J.—SM
Morse, R. V.—M
Murphy, R. B.—A
Murtagh, J. B.—S
Nelson, A. G.—A
Olah, Joseph—A
Osder, S. S.—M
Packer, Leroy—M
Padovano, R. J.—M
Papas, John—A
Parkins, G. J.—M
Pearsall, C. H., Jr.—M
Pelaprat, S. A. L.—S
Perliss, R. E.—M
Perlmutter, A. M.—M
Perry, D. P.—M
Persh, H. R.—S
Peters, G. J.—A
Peterson, H. O.—F
Phagan, R. J.—M
Pighi, L. J.—M
Poppe, C. W.—A
Posner, Solomon—M
Price, David—M
Purdy, R. L.—A
Prichard, J. S.—M
Raber, Samuel—M
Ravner, S. M.—S
Rearwin, R. H.—A
Rehberg, C. F.—A
Roberts, Albert—M
Rosen, Meyer—SM
Rosenthal, S. A.—M
Russell, J. A., III—M
Saarts, Helmut—SGA
San Giovanni, Carlo, Jr.—M
Sant Angelo, M. A.—M
Sayer, George—A
Schiller, H. H.—SM
Schimsky, David—S
Schindwolf, Rudolph—A
Schneider, R. M.—SM
Schneiderman, William—M
Schroeder, K. R.—M
Schulkind, Donald—A
Schwartz, Bernard—M
Schwartz, R. H.—M
Scott, J. E.—A
Seckler, P. J. A.—M
Selnick, L. L.—A
Selvaggio, N. J.—M
Sharp, B. M.—M
Shergalis, D. J.—M
Shulman, Irving—A
Siegel, Robert—S
Sieminski, Edward—SM
Simonelli, N. A.—A
Simonton, L. J.—M
Skwarek, F. J.—SM
Slysh, Paul—M
Smilowitz, S. N.—A
Smith, H. H.—M
Soboleski, W. P.—M
Spence, H. W.—M
Stamler, Leo—SM
Stein, D. B.—A
Steinberg, C. A.—M
Stephenson, J. G.—SM
Sudinsky, Leon—A
Taboski, A. F.—M
Takach, Albert—M
Taylor, J. A.—M
Taylor, J. M.—M
Teltscher, E. W.—M
Thomson, H. C.—M
Tierney, T. J.—M
Tricamo, V. A.—M
Trunk, E. G.—APG
Tucker, A.G.—A
Vinarub, M. E.—M
Vogel, Erwin—SM
Waldrop, J. E., Jr.—A
Walker, A. C.—M
Wallace, R. A.—M
Warren, S. D.—A
Wathen, R. L.—M
Weiner, George—A
Weintraub, Irving—A
Weiss, Gerald—SM
Werst, M. C.—M
Westover, T. A.—SM
Wheeler, H. A.—F
Wiesner, Leo—M
Wild, J. J.—M
Winzemer, A. M.—SM
Witmer, F. S.—M
Witt, C. J.—M
Wurman, Gustave—M
Young, V. J.—SM
Zadoff, S. A.—A
Zetkov, G. A.—M

New York

Albanese, A. P.—M
Alexandro, F. J., Jr.—M
Alloggiamento, Attilio—SGM
Altmark, Seymour—S
Arcand, A. T.—M
Archbald, R. W.—SM
Arguello, R. J.—M
Armstrong, R. W.—SM
Banner, Leonard—M
Barker, D. R.—M
Bastow, J. G., Jr.—M
Baum, M. C.—M
Baumann, D. A.—A
Baxter, D. W.—A
Beattie, J. W.—M
Beaver, M. W.—M
Bellantoni, J. F.—M
Berk, James—M
Berl, Siegmund—M
Berman, L. R.—SGM
Bertram, J. E.—M
Bickel, H. J.—M
Bigelow, S. C.—M
Blaustein, P. H.—M
Blecher, Franklin—M
Bolton, Arthur—M
Boorstyn, R. R.—SGM
Boyd, W. L.—A
Brailey, M. L.—M
Brendle, T. A.—A
Broman, H. F.—M
Burke, L. E.—S
Carini, R. P.—S
Cattelona, J. A.—M
Cerasoli, R. T.—SGA
Chang, S. S. L.—SM
Chien, K. L.—A
Clemens, G. J.—M
Cohen, L. B.—M
Cohen, Stanley—S
Cohn, M. R.—A
Conte, J. A.—SGA
Cranston, R. B.—M
Cressey, J. J. J.—S
Crowley, T. J.—M
Cumming, M. J.—M
Cypser, R. J.—M
D'Amato, R. J.—SM
Davis, E. M., Jr.—SGM
Defilippis, L. S. M.—M
Defloria, R. N.—A
De Lessio, Noel—S
Deptulski, V. V.—S
Deutsch, Simon—A
Diamond, J. E.—M
Diebold, J. T.—A
Di Santi, Nicholas—M
Di Stefano, John—S
Dolkas, Constantine—A
Drossman, M. M.—SGM
Duffy, J. J.—A
Edwards, Andrew, Jr.—A
Erbrecht, G. F.—S
Feinerman, B.—M
Fifer, R. S.—SM
Fink, S. B.—S
Finke, H. A.—M
Fleisher, Harold—M
Freed, Arthur—A
Friedensohn, George—A
Freudenberg, Boris—SM
Frey, J. J.—M
Friedland, Bernard—M
Funk, H. L.—M
Garg, J. M. L.—S
Garofalo, V. J.—S
Garson, E. L.—A
Gilman, G. W.—F
Glantz, L. M.—SM
Glick, S. M.—S
Glomb, J. D.—S
Goetz, Herbert—M
Golden, Donald—M
Goldsmith, A. N.—F
Goodman, L. J.—S
Gottlieb, R. M.—M
Gouyet, J. J.—M
Grayson, L. P.—S
Green, B. W.—SGM
Greenberg, E. P.—M
Groginsky, H. L.—M
Gronner, Alfred—SM
Grossman, Robert—M
Guenther, Richard—M
Guardino, R. C.—M
Guthait, Manuel—A
Haddad, R. A.—S
Harder, Dorothea—M
Harries, Wolfgang—M
Harty, L. T.—A
Hastings, J. J.—M
Heller, Samuel—M
Hauerstock, Conrad—S
Ho, K. W.—S
Hodge, Bartow—M
Hough, J. B.—A
Jorysz, Alfred—M
Kaplan, K. R.—M
Kassel, Aaron—A
Katz, M. D.—M
Katz, R. A.—S
Kiel, J. H.—A
Kline, B. H.—M
Knapp, J. Z.—M
Koepcke, R. W.—A
Korrol, C. R.—M
Ku, W. H.—S
Kwo, T. T.—M
Kyle, D. W.—SGM
Lampert, Leon—M
Lannary, John—M
Lazarus, P.—M
Lee, S. P.—S
Liebman, P. M.—M
Lindner, N. J.—M
Liu, Bede—S
Low, Frank—M
Maedel, G. F.—SM
Mahrous, Haroun—SM
Malina, Meyer—M
Marcinkowski, H. L.—M
Marshall, S. L.—A
Martin, R. R.—SM
Martinson, F. L.—SM
Massell, Edward—SM
Miller, Barry—M
Modugno, V. D., Jr.—SGA
Molinelli, R. J.—M
Moskowitz, I. W.—S
Moyer, J. J.—M
Mujica, T. H.—S
Newman, Leonard—M
Novick, William—M
Ortmann, M. W.—S
Parr, A. F. W.—M
Paschetto, E. J.—M
Perry, Lawrence—M
Piore, E. R.—F
Plasters, G. T.—S
Porter, R. W.—SM
Prager, Saul—S
Preiss, R. J.—A
Putterman, Harry—M
Quan, Stanley—S
Ragazzini, J. R.—F
Reeves, J. F.—A
Reynolds, Gibson—A
Riley, W. B.—M
Roberts, R. P.—M
Rosenberg, H. J.—A
Rosenthal, Lee—S
Rothenstein, Stuart—S
Royer, G. H.—M
Rudofsky, B. I.—M
Rugge, R. A.—SM
Saddeh, David—M
Sakson, P. J.—M
Sales, Murray—M
Sandock, Jules—A
Sarachik, P. E.—S
Schischa, Erwin—S
Schoenfeld, R. L.—M
Schwartz, J. H.—M
Schwarz, R. J.—SM
Shapiro, Eugene—M
Shapiro, Paul—M
Sherman, Seymour—M
Shooman, M. L.—M
Sigona, J. F.—M
Silvera, R. V.—SGM
Sinclaire, F. S.—M
Skalski, Julius—S
Slavin, M. J.—M
Soreny, E. V.—A
Steinman, G. R.—S
Steinmetz, Henry—S
Sterman, I. I.—M
Suarez, Anthony—A
Thau, F. E.—S
Thomas, R. O.—M
Truxal, J. G.—SM
Turchiano, M. W.—A
Turczyn, W. A.—M
Ur, Hanoch—M
Vieira, J. F.—A
Vigants, Arvids—S
Von Hoyer, Andre—S
Walton, J. S. V.—A
Watkins, J. E.—M
Weisman, Irving—M
Weiss, A. J.—S
Wernick, J. I.—A
Wertheim, L. A.—M
Willett, Edward—M
Winkelstein, R. A.—M
Wolfson, Richard—M
Wolin, Walter—S
Wright, Donald—S
Young, K. P.—M
Young, R. O.—M
Yu, E. K. C.—M
Zadeh, L. A.—SM
Zechnowitz, A. L.—M
Zourides, V. G.—M

Northern New Jersey

Aaron, M. R.—M
Ackley, J. N.—M
Aiello, Michael—M
Alexander, E. J.—M
Anderson, N. E.—M
Antonazzi, F. J.—M
Bahls, W. E.—SM
Bailey, E. M.—A
Barnett, W. T.—M
Bearman, A. L.—A
Bodnar, E. R.—S
Bogner, Irving—SM
Bowker, M. W.—A
Brown, A. T., III—A
Brown, R. I.—M
Byrne, E. R.—M
Cater, J. R.—SM
Corradi, F. J., Jr.—A
Cowles, W. W.—S
Cynamon, J. J.—A
Davis, E. S.—M
De Franco, J. J.—M
Digrindakis, Michael—M
Di Massimo, Donald—S
Doba, Stephen, Jr.—SM
Dolinsky, G. M.—S
Doniger, Jerry—M
Earnshaw, E. F., Jr.—M
Eckenfelder, R. C.—M
Egan, T. R.—M
Elwell, H. G., Jr.—M
Engelberg, Charles—M
Finkelstein, David—M
Forsythe, A. M.—SGM
Frimtzis, Robert—M
Fromer, Morton—M
Gardner, J. L.—A
Genzel, Stephen—SGA
Goldberg, S. R.—M
Goodman, H. C.—M
Gordon, M. J.—M
Grandmont, P. E.—M
Hamilton, R. D.—SM
Hamming, R. W.—A
Hanselman, K. J.—M
Hilty, D. C.—SGM
Humphrey, R. M.—A
Hunter, W. B.—A
Johnson, L. C.—A

Kaszerman, Philip—M
Kierce, Eugene—S
Kowalczyk, R. R.—A
Kozimor, Marian—M
Kuehler, H. R. M
Kulik, V. A.—M
Kunkel, E. A., Jr.—M
Kurczewski, R. V.—S
Lawrence, C. W.—S
Lazos, N. J.—A
Leeds, Myron—A
Losher, M. I.—A
Louis, R. A.—M
Lozier, J. C.—SM
Luce, C. D.—S
Lunney, R. E.—M
Malone, Martin—M
Mandel, Mark—M
Mathews, M. V.—A
Mayo, J. S.—A
McCrory, J. R.—M
McIntyre, J. W.—A
Meinholtz, H. J.—A
Meyers, S. J.—M
Morrow, S. R.—A
Mount, Ellis—M
Mueller, P. L.—M
Mulligan, J. H., Jr.—SM
Murray, A. A.—S
Newhall, E. E.—A
Nielsen, D. M.—APG
O'Neill, M.P.—M
Oseas, Jonathan—M
Ossanna, J. F., Jr.—M
Otis, A. N., Jr.—A
Panter, P. F.—SM
Paradise, R. Y.—M
Perlis, H. J.—M
Peters, D. P.—SGM
Podell, R. L.—A
Potash, Jerome—M
Reilly, R. A.—A
Ricci, R. J.—S
Richards, G. P.—M
Robinson, A. S.—M
Rosenthal, C. W.—M
Rosenthal, M. S.—M
Rossi, Stephen—A
Russell, F. A.—SM
Schnall, Emanuel—M
Schwartz, A. M.—A
Shangraw, C. C.—SM
Sippach, F. W., Jr.—M
Slana, M. F.—SGM
Smith, E. J.—M
Solomon, R. H.—SGM
Stickle, R. L.—A
Stiefel, K. E.—M
Sudduth, W. B.—A
Sutton, L. E., III—A
Sutton, R. H.—A
Thomas, D. E.—SM
Thompson, C. F.—A
Tighe, D. J.—S
Torrey, L. W., Jr.—M
Townsend, R. L.—A
Unger, Arnold—M
Van Hoff, P. A.—S
Veazie, Waldemar—SGM
Waddell, R. G.—S
Warden, F. W.—SM
Wetzel, K. C.—A
Wilde, A. E., Jr.—A
Willes, R. L.—A
Winter, J. W.—A
Winter, R. A.—M
Wolfe, T. R.—S
Woodman, R. A., Jr.—M
Worhach, Russell—M
Zayac, F. R.—A

Princeton

Amarel, Saul—SM
Aronson, A. I.—M
Beck, G. A.—S
Chaykowsky, O. C.—M
De Versterre, W. I.—M
Downie, D. E.—M
Faustini, Carlo—M
Fernald, O. H.—M
Garretson, E. B.—A
Graber, G. F.—S
Gross, Josef—M
Helms, H. D.—S
Hoedemaker, R. W.—M
Kang, C. L.—M
Krittman, I. M.—M
Lemelson, J. H.—A
Maitra, K. K.—M
Norton, J. A.—S
Pressey, C. W.—A
Radvany, I. E.—S
Rogers, A. E.—M
Ruble, G. B.—M
Schofield, C. R.—A
Schorr, Herbert—M
Sklansky, Jack—SM
Smith, Beresford—M
Staffin, Robert—A
Surber, W. H., Jr.—M
Szwedo, S. G.—A
Thompson, T. H.—M
Truitt, T. D.—S
Westneat, A. S., Jr.—SM

Region 3

Atlanta

Ackerman, C. L.—S
Bohr, E. T.—M
Bradfute, G. A., Jr.—A
Chambers, J. W., Jr.—S
Dobson, W. A., Jr.—S
Fry, J. C.—S
Mehaffey, J. H., Jr.—S
Raymond, R. E.—S
Robertson, D. W.—SM
Smith, G. A., III —S
Steves, R. B.—S
Ziegler, N. F.—A

Baltimore

Aiken, R. T.—S
Axelby, G. S.—SM
Baida, Seymour—M
Baker, D. A.—M
Barlow, H. C.—SM
Barnett, Leon—SGM
Barrack, C. M.—A
Barton, J. H.—M
Beckman, N. W.—A
Behm, G. T.—SGM
Bentley, F. C.—M
Burns, J. E.—M
Choksy, N. H.—M
Cichanowicz, H. J.—A
Coulter, E. L.—M
Eaton, T. T.—SM
Edwards, R. L., Jr.—SGM
Esterson, G. L.—M
Farkas, Joseph—A
Fegely, W. D.—M
Fischer, P. P.—M
Floam, J. S.—A
Gambrill, R. D.—M
Gessner, Urs—A
Glaser, E. M.—M
Glover, C. C.—M
Goulden, D. E.—A
Groszer, A. J., Jr.—A
Hauf, J. C., III—A
Horn, R. E.—A
Huddleston, F. J.—SM
Hurley, W. A.—M
Jackson, J. H.—M
Jenkins, J. L.—M
Jentilet, Adam—M
Jones, L. G. F.—SM
Kalman, R. E.—A
Kegel, A. G.—SM
Kernan, Paul—SM
Koziara, E. H.—M
Leahy, F. N.—A
Lee, Marshall—S
Longuemare, R. N., Jr.—M
Meador, A. B., Jr.—M
Miller, W. H., Jr.—M
Mortimer, T. S.—APG
Osborne, E. F.—M
Owra, W. M.—SGM
Penabad, Joseph, Jr.—A
Pincoffs, P. H.—M
Plath, R. H.—M
Poswiatowski, W. C., III—A
Pulitano, F. J.—M
Ralston, George—M
Ramsay, J. H.—M
Raymond, W. F.—M
Raynes, H. D.—M
Reel, G. M.—M
Rogers, C. L., Jr.—A
Rosenberry, W. W.—SM
Ryan, C. C.—M
Sander, W. E.—M
Sanford, R. W.—M
Seldon, W. S.—A
Slavin, Michael—A
Spink, P. G.—A
Stebbins, W. J.—M
Stephenson, J. O.—M
Strull, Gene—M
Taragin, Saul—M
Thomas, J. F.—M
Visher, W. A.—A
Walters, J. L.—M
Weems, C. M., Jr.—M
Whitcomb, M. F.—A
Wilson, H. C.—M

Central Florida

Alexander, J. E.—M
Bendett, R. M.—M
Bridgland, T. F., Jr.—M
Buchan, J. F.—A
Dibble, H. L.—M
Duval, A. N.—A
Elgerd, O. I.—SM
Ferguson, R. E.—A
Fisk, N. H.—M
Gray, A. R.—SM
Hackert, William, Jr.—A
Houston, E. E., Jr.—M
Kasowski, S. E.—M
Koning, R. E.—M
Lanzkron, R. W.—M
Le Gare, J. M.—SM
Meara, V. T.—M
Painter, Parker, Jr.—SM
Parker, R. L., Jr.—SGM
Pelchat, G. M.—M
Powers, D. M.—M
Rhodes, D. R.—SM
Rose, R. M.—S
Stifler, W. W., Jr.—SM
Stout, J. R.—S
Stringer, K. E.—S
Walker, J. R.—M
Young, W. L.—M

Florida West Coast

Adkisson, W. M.—M
Ainsworth, F. W.—M
Colbert, D. C.—M
Dingley, E. N., Jr.—F
Dowell, W. B., Jr.—M
Goldie, S. P.—M
Hoffman, R. S.—M
Inman, T. F.—SM
Landry, W. J.—SM
Loebel, M. B.—C
Macomber, G. R.—A
Parry, J. O.—A
Parvin, R. H.—M
Rosenzvaig, J. E.—M
Saraydar, R. A.—M
Uglow, K. M., Jr.—SM
Wells, E. L.—A

Huntsville

Bradley, B. C.—M
Christopher, J. P.—SGM
Davis, J. C.—S
Dunkel, H. D.—M
Edling, E. A.—A
Feaster, W. M.—SGM
Gentry, E. B.—M
Maddox, W. V.—S
Norman, C. F.—M
Otto, W. F.—M
Pittman, W. C.—M
Porter, S. N.—A
Pouncey, A. D.—SGM
Salonimer, D. J.—M
Schwab, W. G.—M
Seashore, C. R.—M
Watanabe, A. S.—M
Young, J. B., Jr.—S

Miami

Cook, H. A.—M
Greenfield, Alexander—SM
Lampkin, G. F.—VA
La Tour, John, Jr.—M

North Carolina

Cronin, H. C.—M
Littleton, W. W.—M
Rathmell, J. E.—SGM
Register, L. H., Jr.—S
Stephens, T. L.—M
Young, D. B.—M

Northwest Florida

Stoffel, B. L.—M

Philadelphia

Affel, H. A., Jr.—SM
Aires, R. H.—M
Alperin, N. N.—A
Anderson, J. M.—S
Anderson, W. G.—A
Auerbach, I. L.—F
Azaren, David—M
Bachofer, H. L.—A
Baker, W. W.—M
Bannar, C. J., Jr.—S
Barger, J. R.—M
Bechtold, G. W.—M
Beck, Cyrus—A
Beck, M. R.—M
Benner, A. H.—SM
Berkowitz, R. S.—M
Bernstein, Fred—A
Beter, R. H.—SM
Blasberg, L. A.—SM
Boreen, H. I.—M
Bradley, R. E.—M
Bradley, W. E.—F
Brown, Irving—M
Brucklacher, J. E., Jr.—M
Bruno, J. L., Jr.—S
Burke, R. H.—SGM
Buxton, P. T.—S
Bycer, B. B.—SM
Campanella, M. J.—M
Canavan, T. P.—M
Capron, R. W.—M
Cecal, J. A.—A
Chronister, W. M.—M
Chudleigh, W. H., Jr.—SM
Cohen, B. H.—M
Colehower, C. H., Jr.—M
Connaught, P. M.—M
Crossen, E. J.—S
Curtin, W. A.—M
Dahlin, E. B.—M
D'Andrea, L. L.—SM
Davidson, R. C.—A
Davis, R. W., Jr.—M
Davis, T. F.—M
Deacon, N. E.—M
Dehm, R. E.—A
De La Cuesta, Hernando—S
Desautels, G. L.—M
Deutsch, Joseph—A
Dickens, B. L.—M
Dordick, H. S.—M
Dymecki, J. W.—S
Epstein, Herman—SM
Fabbioli, L. F.—M
Farkas, Leonard—M
Farrelly, R. J.—A
Fath, J. P.—A
Faust, A. C.—M
Fegley, K. A.—M
Fenton, F. H., Jr.—M
Fey, R. C.—S
Fierstein, J. H.—A
Fischbeck, K. H.—A
Fogelberg, A. E., Jr.—SM
Foley, G. M.—SM
Forte, S. R.—M
Frank, J. S.—M
Friend, A. W.—F
Fuchs, A. M.—M
Fujimoto, Akira—M
Gallagher, G. J.—A
Gardiner, F. J.—SM
Glassman, Irving—M
Gluck, S. E.—M
Goff, K. W.—A
Gottschalk, J. M.—M
Gravatt, R. C.—S
Greely, D. K.—A
Greenfield, Milton—A
Gregory, T. R.—M
Griep, D. J.—S
Grim, E. D.—M
Halket, D. R.—M
Halsted, C. P.—A
Hammond, J. M.—S
Hartnett, E. J.—M
Hellerman, Herbert—M
Herrmann, J. E.—M
Hitt, J. J.—M
Hollander, G. L.—SM
Hoover, E. W.—M
Howell, Monroe—M
Hughes, P. F.—S
Huyett, W. I.—M
Isom, W. R.—SM
Kanal, L. N.—A
Karew, J. J.—M
Kass, Sholom—M
Koblenz, A. M.—M
Kolodner, Meyer—M
Kozikowski, J. L.—M
Krantz, F. H.—M
Ku, Y. H.—SM
Laskey, J. M.—M
Lathrop, R. A.—M
Laurent, G. J.—SM
La Verghetta, F. E.—SGM
Lazarus, F. F.—M
Lazinski, R. H.—M
Levin, Robert—SGM
Levy, A. S.—M
Lieb, A. B.—A
Linhardt, R. J.—SM
Lisicky, A. J.—M
Lockhart, J. C.—M
Loev, David—A
Loftus, J. A.—M
Lovett, R. S.—M
Luoto, U. A.—M
Lynch, J. T.—M
Macqueene, P. H.—SM
Maguire, H. T.—A
Markarian, Hagop—A
Marks, T. P.—A
Marra, Thomas—M
Mayhew, T. R.—M
McClure, R. W.—A
McWilliams, C. R.—M
McWilliams, G. R.—A
Miller, G. F.—M
Moll, W. H.—M
Murr, R. H.—S
Neumann, L. R.—A
Nye, D. D., Jr.—M
O'Brien, J. F., Jr.—M
Osbahr, B. F.—SM
Paskman, Martin—M
Perecinic, W. S.—M
Porter, J. W.—M
Potosky, Maurice—M
Preston, G. W.—SM
Price, J. F.—M
Rangachar, H. V.—S
Rao, G. V.—M
Rawdin, Eugene—A
Reich, A. L.—M
Renfrow, R. J.—M
Richter, Filmore—M
Riggs, D. L.—M
Rogers, D. A.—M
Rogers, R. F.—A
Roop, R. W.—SM
Rudnick, J. J.—A
Rudofsky, Samuel—A
Ruf, F. C.—SGM
Rutledge, J. E., Jr.—M
Sapp, D. H.—M
Schreiner, W. A.—M
Seawell, W. N.—M
Selinsky, J. J.—M
Sepahban, A. H.—A
Sevian, E. A.—M
Sheffler, W. J.—M
Shelton, R. M.—M
Shih, Wei-Ming—M
Shucker, Sidney—SM
Shulman, Louis—M
Sink, J. A.—A
Skavicus, A. J.—A
Skinner, N. W.—A
Smith, D. B.—F
Sobolewski, Frank—M
Sorkin, C. S.—A
Spindler, C. W., Jr.—SM
Sposato, B. J.—S
Steinberg, B. D.—A
Strauss, H. E.—M
Strip, Joseph—SM
Stubbs, G. S.—M
Sun, H. H.—A
Swartz, M. D.—A
Tabor, C. J., Jr.—M
Taenzer, Erwin—SM
Tonooka, Shinichi—M
Trout, H. J., Jr.—A
Wagner, R. J.—S
Walker, H. R.—A
Weiner, J. R.—SM
Weisenberger, A. J.—M
Weiss, Eric—M
Williams, R. J., III—M
Wirtz, E. L.—A
Wolfson, H. S.—M
Wolin, Louis—SM
Woodcock, V. E.—M
Wooten, L. B.—A
Wykstra, E. A.—S
Yang, Tsute—M

South Carolina

Brooks, M. J.—M
Glaser, H. I.—M
Pippin, R. F., Jr.—M

Virginia

Andrews, R. E.—M
Black, J. H.—A
Branscom, G. A.—M
Burlingame, C. W.—A
Cockrell, W. D.—SM
Dial, E. W.—M
Fourdriat, E. C.—APG
Gehrke, W. C.—SGM
Gregory, C. A., Jr.—M
Harvey, G. L.—SM
Passera, A. L.—M
Pettus, W. W., IV—M
Spitzer, D. M., Jr.—S
Welch, A. A.—M
Zaluski, J. P.—M

Washington, D. C.

Aaron, J. B., Jr.—SGM
Allen, D. A.—A
Antonov, A. F.—A
Barbeau, A. R.—M
Bass, C. A.—M
Britner, R. O., Jr.—M
Burgess, A. G.—M
Bush, A. G., Jr.—M
Bush, G. B.—M
Chu, Yaohan—M
Clarke, A. S.—SM
Collins, J. R.—M
Connelly, E. M.—A
Coyle, R. J.—A
Davies, G. L.—SM
Davis, T. R.—A
Diels, J. C.—M
Dillon, J. C., Jr.—M
Duning, K. E. W.—M
Enos, R. M.—M
Finkel, Abraham—M
Fischer, J. R.—M
Fleming, J. J.—M
Fox, A. L.—S
Gale, Morten—A
Gaylord, R. E.—M
George, S. F.—SM
Godsey, W. J.—M
Gorozdos, R. E.—M
Greenberg, D. L.—M
Hargraves, J. F.—S
Heil, M. M.—S
Herbert, T. O.—M
Hibbard, W. D., Jr.—M
Hocking, L. J.—A
James, W. G.—SM
Karp, M. A.—SM
Kirshner, J. M.—A
Lee, F. M.—A
Lee, J. D.—M
Little, J. L.—M
Looney, C. H., Jr.—A
Mallin, J. A.—M
Mischler, E. F.—SM
Misner, R. D.—SM
Mitchell, G. J.—M
Morrisey, J. A.—A
Nelson, M. W.—A
Nickell, W. C.—S
O'Hara, J. J., Jr.—M
Ostaff, W. A.—SM
Penniman, I. R.—M
Pike, W. N.—M
Poor, V. D.—M
Prival, H. G.—SGM
Ramos, Edward—M

Rawling, A. G.—APG
Reagen, E. J.—M
Regardh, C. B.—A
Roberts, G. L., Jr.—S
Rogers, A. L.—A
Rolinski, A. J.—A
Rozanski, R. R. A.—M
Scott, D. G.—S
Shapiro, Gustave—SM
Shaw, J. S.—S
Shepard, D. H.—SM
Smith, W. R., III—S
Sokolove, F. L.—S
Spigelthal, E. S.—M
Spool, James—M
Stoops, C. W.—M
Talkin, A. I.—M
Van Lunen, R. D.—M
Viera, Frank, Jr.—A
Waterman, Peter—M
Watkins, P. L.—M
White, C. F.—M
Wilcomb, E. F.—SM
Wimmer, P. L.—M
Zastrow, K. D.—A

Region 4

Akron

Barnett, C. B.—M
Brown, D. L.—A
Buxton, A. C.—SM
Chimera, V. J.—SM
Colletti, Nicasio—M
Cook, E. E.—M
Diamantides, N. D.—M
Flowers, H. L.—SM
Haas, D. L.—SM
Hann, D. D.—A
Hermann, P. J.—SM
Ingalls, R. S.—M
Lambert, C. O.—SM
Lanier, H. F.—SM
Meilander, W. C.—M
Miller, J. H.—M
Oldham, D. K.—S
Ryburn, P. W.—M
Stringer, R. B.—M
Toman, W. J. V.—M
Yarosh, N. P.—SM
Yochelson, Saul—M

Central Pennsylvania

Bacchialoni, F. L.—M
Harvey, H. B.—M
Knausenberger, G. E.—M
Lawther, J. M.—A
Oblinger, J. T.—M
Seeley, R. M., Jr.—M
Wolfe, R. E.—M

Cincinnati

Berg, D. F.—M
Bernhard, C. J.—A
Besco, F. E., Jr.—M
Burnett, L. A.—M
Culy, G. E.—S
Dale, W. L.—A
Doerr, W. H.—M
Ehlers, M. W.—A
Engelmann, R. H.—SM
Georger, L. J.—M
Hellen, R. J.—M
Herrin, C. B.—M
Hulstrand, B. E.—M
Keene, L. C.—SM
Marhauer, H. H.—S
Meuleman, Robert—SM
Mohr, P. A.—M
Nistico, Frank—M
Thomas, J. P.—M
Westmark, J. E.—A
Zupansky, Milo—A

Cleveland

Bolz, R. W.—SM
Chevalier, C. C., Jr.—S
Clutz, D. A.—S
Dambach, R. A.—SM
Gogia, J. K.—A
Grasson, Walter, Jr.—M
Haner, Lambert—A
Hart, C. E.—A
Hotchkin, E. E.—M
Karkalik, F. G.—S
Kinkaid, J. C.—A
Kliever, W. H.—SM
Klock, H. F.—M
Levisay, D. H.—M
Louis, J. R.—M
Majercak, J. V.—M
Miller, H. J.—M
Miro-Nicolau, Jose—S
Murray, J. E.—M
Nadkarni, D. D.—S
Saltzer, Charles—M
Shepherd, B. R.—M
Wiik, Erik—S

Columbus

Bain, T. D., Jr.—A
Bishop, A. B., III—M
Bjerre, Paul—M
Bornhorst, K. F.—M
Burgener, R. C.—A
Butterworth, G. S.—M
Chope, H. R.—SM
Cohen, Donald—M
Conlon, R. J.—M
Cosgriff, R. L.—SM
Coulter, N. A., Jr.—APG
Cummins, M. M.—S
Del Favero, R. J.—S
England, S. J. M.—A
Goldman, N. I.—S
Gulcher, R. H.—M
Johnson, J. E.—S
Kirschbaum, H. S.—A
Leonard, J. D.—SM
Lewis, D. E.—M
McFarland, R. S.—A
Mobley, M. D.—S
Moll, Magnus—A
Pifer, P. M.—M
Sapp, E. R.—S
Studtmann, G. H., Jr.—M
Tamplin, G. R.—M
Weimer, F. C.—M

Dayton

Curtis, L. L.—A
Feldmann, R. J.—S
Fenton, R. E.—M
Fulton, W. L., Jr.—M
Graham, R. L.—SGM
Grimm, F. W.—A
Kerbs, W. A.—A
Kiebert, M. V., Jr.—SM
Martino, J. P.—M
Morgan, B. S., Jr.—SGM
Peterson, L. S.—M
Sansom, F. J.—M
Simopoulos, N T.—A
Thomas, E. R.—M
Thompson, J. P.—M
Toomey, P. J.—M
Wolaver, L. E.—A

Detroit

Amber, P. S.—M
Barcus, Ronald—M
Bartek, R. J.—M
Becher, W. D.—M
Benaglio, R. V.—M
Blackwell, W. A.—M
Brabant, C. E.—A
Brown, L. R.—A
Brunais, E. G.—A
Bublitz, A. T.—A
Burr, Harold—S
Buxton, C. E.—M
Chaney, L. W.—M
Chuang, Kuei—M
Chute, G. M.—SM
Epley, D. H.—S
Fife, D. W.—M
Foulke, J. A.—SGM
Frese, R. E.—A
Gaskill, R. A.—M
Gilbert, E. G.—M
Gilbert, E. O.—M
Harding, W. G.—M
Hartford, T. W.—M
Jackson, H. L.—S
Johnson, S. M.—A
Karwas, F. C.—M
Klem, R. F.—SM
Klimek, T. F.—SGM
Kobayashi, H. S.—SGM
Lindahl, C. E.—S
Lovalenti, Sam—M
Martin, T. A.—S
McGlinn, E. J.—M
McKelvey, E. J.—M
McKelvie, J. L.—M
Nakagawa, Noriyuki—A
Nauts, D. F.—SGM
Nixon, J. D.—S
O'Neal, R. D.—SM
Patton, H. W.—SM
Pulliam, P. E.—M
Rauch, L. L.—SM
Rehwoldt, T. V.—M
Retko, Edward—A
Robinson, D. F.—M
Robinson, G. H.—M
Roth, P. F.—M
Seleno, A. A.—M
Smith, B. A.—M
Smith, D. F.—A
Smith, Wray—M
Strand, John—M
Sutton, W. A.—A
Taplin, L. B.—M
Tokad, Yilmaz—A
Turkish, M. C.—M
Wallace, V. L.—M
Webber, R. C.—S
White, Ozelle—M
Whiteside, A. E.—M
Willett, R. M.—M
Zuroff, C. A.—M

Emporium

Carlson, J. L.—SGM
Little, D. R.—M
Wright, J. B.—A

Pittsburgh

Aronson, M. H.—M
Bhavnani, K. H.—M
Blewitt, D. D.—M
Bossart, P. N.—F
Boyd, J. J.—S
Byerly, R. T.—M
Chang, H. K. H.—M
Chen, Kan—A
Cilyo, F. F., Jr.—M
Coates, R. S.—A
Coccia, R. A.—SGM
Eggers, C. W.—S
Ellison, B. P.—M
Feldman, J. M.—S
Ford, D. J.—M
Golla, E. F.—SM
Mathias, R. A.—SM
Meindl, J. D.—S
Moss, A. I.—M
Mucci, Geno—A
Murray, D. W.—A
O'Donnell, J. J.—M
O'Shea, R. P.—S
Rathbone, D. E.—M
Rau, F. J.—M
Robl, R. F., Jr.—A
Rogers, L. J.—M
Schindler, D. G.—M
Schwindt, A. J.—M
Shou, S. H.—S
Spriggs, L. A., Jr.—M
Sze, T. W.—A
Thompson, F. T.—A
Varhola, E. M.—S
Wolford, J. E.—A

Toledo

Dinning, J. R.—M
Ewing, D. J., Jr.—M
Fuller, L. E.—A
Krieger, A. E.—A
Leuck, D. D.—M
Murley, E. M., Jr.—M

Western Michigan

Cummins, D. L.—M
Mort, V. A.—A

Williamsport

Michaels, G. J.—SGM
Webb, H. E.—M
Young, G. D.—A

Region 5

Cedar Rapids

French, D. W.—S
Girard, L. J.—M
Golden, R. W.—S
Hedgcock, W. T., Jr.—M
Keser, P. D.—M
Lowenberg, E. C.—M
Luik, Ilmar—M
Pekarek, K. L.—A
Quinn, J. A.—S
Swinton, S. Q.—S

Chicago

Ahring, F. E.—A
Auth, L. V., Jr.—M
Axelrod, L. R.—M
Bielenin, Andrew—A
Bilsens, Gunars—S
Blyth, R. A.—M
Borkovitz, H. S.—M
Boyd, D. M., Jr.—SM
Bullen, C. V.—M
Bulliet, L. J.—A
Burtness, R. W.—SM
Cannon, C. D.—M
Carter, Robert—A
Chorney, P. L.—M
Chulsky, Isadore—M
Costa, P. J.—SM
Cruz, J. B., Jr.—M
Deterville, R. J.—M
Dikinis, D. V.—M
Drane, R. L.—M
Druz, W. S.—SM
Dryden, R. C.—S
Dumay, E. C.—A
Ebstein, Bernhard—M
Egan, J. F.—S
Eilers, C. G.—M
Engelland, G. C.—A
Epley, D. L.—S
Even, J. C., Jr.—SGM
Eksten, D. G.—M
Falk, W. R.—M
Ferre, G. E.—M
Fett, G. H.—SM
Fu, King-Sun—M
Foster, G. E.—M
Ghosh, H. N.—SGM
Givens, S. D.
Glyptis, Nicholas—A
Greenberg, C. J.—M
Gregory, E. C.—SM
Hamilton, Sanborn—A
Hansen, T. A.—SM
Holman, W. J.—SM
Hutson, D. E.—S
Jenness, R. R.—SM
Johnson, T. L.—S
Jones, R. W.—SM
Kabrisky, M. J.—M
Kantner, H. H.—M
Karres, G. E.—M
Kott, W. O.—SM
Kuo, B. C. I—M
Kreer, J. B.—A
Lafferty, V. C. -M
Laney, B. H.—S
Leslie, J. D.—M
Lesnick, Robert—SM
Lewis, H. A.—M
Lewis, P. H.—M
Li, Ching-Chung—S
Ma, H. J.—S
Maddox, C. L.—S
Maginot, J. J.—M
Martin, J. W., Jr.—M
Merrifield, L. A.—M
Meyer, Andrew—A
Mittelmann, Eugene—F
Moe, M. L.—S
Murphy, G. J.—M
Myers, B. R.—SM
Nelles, Maurice—M
Nelson, G. L.—M
Oldham, K. W.—M
Paterson, W. L.—A
Pugsley, J. H.—S
Reukauf, D. C.—M
Riley, T. M.—S
Rockwood, C. C.—A
Sayles, H. L.—A
Shewan, William—M
Sommeria, M. R.—A
Spademan, C. F.—M
Stirling, R. C.—S
Thake, R. F.—M
Thomas, R. G.—A
Toepfer, R. E., Jr.—S
Unitis, E. R.—A
Van Bosse, J. G.—A
Van Ness, J. E.—SM
Van Valkenburg, M. E.—SM
Verbanec, W. R.—M
Vitous, J. P.—A
Wallman, E. J., Jr.—M
Warshawsky, Jay—M
Watson, R. B.—SGA
Weiherman, R. J.—SGM
White, E. S.—SM
Wishner, R. P.—S
Ye, J. W. F.—SGM
Zaremba, R. E.—SGM
Zaverdas, N. G.—S

Evansville-Owensboro

Jordan, J. D.—M
Rickerson, W. B.—M

Ft. Wayne

Garrett, L. A.—M
Richeson, W. E.—M
Stein, E. F.—M

Indianapolis

Austin, G. A.—M
Chassin, C. A.—M
Cross, K. R.—A
Evans, R. A.—SM
Galloway, F. M.—S
Gibson, J. E.—M
Grossnickle, R. L.—SGM
Israel, J. D.—S
Kenyon, R. R.—M
Kinnen, Edwin—A
Landgrebe, Dave—M
Meditch, J. S.—S
Miller, D. C.—A
Ogborn, L. L.—A
Raible, R. H.—SGM
Ryan, A. C.—S
Ryan, M. C.—SGM
Sabbagh, H. A.—S
Sage, A. P., Jr.—M
Skinner, G. M.—M
Stanley, P. E.—M
Traznik, E. G.—SGM
Tou, Julius—SM
Van Ostrand, W. F.—S
Whitaker, R. O.—A
White, S. A.—S
Windsor, R. N.—A
Winton, H. L.—SGM
Young, H. S. W.—S

Louisville

Ebaugh, D. P.—A
Erdman, B. K.—M
Lahr, R. J.—SGM
Whitfield, J. E., Jr.—S

Milwaukee

Ackmann, J. J.—S
Bennett, T. H.—S
Berkovec, J. W.—A
Bourbeau, F. J.—A
Boutelle, J. O.—A
Brandt, R. T.—S
Briedis, Gunars—A
Carlson, A. W.—M
Cork, H. A.—M
Doyen, R. G.—A
Eckel, J. R., Jr.—M
Frankos, D. T.—M
Gmeiner, G. T.—SGM
Goggio, E. C.—SM
Graham, J. D.—A
Gruzinski, R. T.—S
Henning, D. A.—SGA
Hill, R. F.—M
Hill, R. G.—A
Jensen, K. S.—SM
Johnson, R. J.—S
Johnson, R. R.—A
Kerske, J. F.—S
Lemere, J. P.—M
Limpel, E. J.—M
Lindemann, A. W.—M
Macherey, R. E.—M
McCumber, R. D.—S
Mertz, R. L.—S
Murrish, C. H.—S
Noble, D. S.—SM
Norum, V. D.—S
Odenweller, R. L.—SGA
Pierce, R. L.—M
Pozorski, J. J.—M
Rekoff, M. G., Jr.—M
Roth, J. E., Jr.—M
Sackett, R. W.—A
Schlicht, R. W.—S
Schumacher, P. J., Jr.—M
Smith, C. C.—A
Trinkner, C. C.—S

Omaha-Lincoln

Allington, R. W.—S
Bashara, N. M.—M
Bollesen, V. P.—SGM
Eklund, R. C.—SGM
Grieninger, J. W.—S

South Bend-Mishawaka

Bymberg, R. J., Jr.—M
Colten, J. L.—SM
Cunningham, G. W., Jr.—M
Ernst, T. W.—S
Hansen, A. G., Jr.—M
Hoffman, C. H.—SM

Twin Cities

Adams, G. E.—SM
Alderson, R. C.—M
Anderson, G. G.—S
Anderson, P. M.—M
Bargen, D. W.—SGM
Bartlett, V. W.—A
Beck, V. W.—M
Benassi, D. A.—M
Bergan, K. N.—SM
Burns, S. S.—M
Finegan, E. F., Jr.—SGM
Geronime, R. L.—A
Gise, F. G., Jr.—A
Glewwe, H. L.—S
Gustafson, H. A.—M
Hardenbergh, G. A.—A
Harvey, P. O.—M
Heiser, R. K.—S
Keel, B. G.—M
Kershaw, J. A.—M
Ketchum, J. R.—M
Kiene, R. C.—M
Knoblauch, Arthur, Jr.—M
Kukuk, H. S.—M
Lahue, P. M.—M
Lode, Tenny—A
Ludwig, J. T.—A
Luh, J. Y. S.—A
Markusen, D. L.—M
McLane, R. C.—A
Moe, W. J.—M
Murray, R. L.—S
Muckenhirn, O. W.—SM
Nellis, W. M.—M
Nordstrom, J. E.—A
Ormsby, R. D.—A
Park, G. L.—S
Pierce, A. J.—A
Prom, G. J.—M
Reed, M. W.—S
Rowland, C. A., Jr.—M
Schuck, O. H.—F
Senstad, P. D.—M
Smith, D. A.—S
Stone, N. T.—A
Storm, J. F.—A
Swanholm, W. J.—SGM
Swanlund, G. D.—S
Thue, B. H.—S
Tsui, Y. T.—A
Tu, J. C.—M
Woll, L. J.—M

Region 6

Dallas

Anderson, R. P.—M
Bailey, R. W.—M
Barnett, M. L.—M
Braginton, P. R.—SM
Brown, K. C.—M
Day, W. J.—M
Dubose, G. P.—M
Faith, W. O.—SM
Grubbs, C. E.—M
Humke, F. O., Jr.—M

Hutson, R. N.—M
Ivy, A. W.—M
Kettler, C. L.—M
McDonald, Marshall—A
McKinney, R. C.—M
Miller, N. D.—SM
Ocnaschek, F. J.—SM
Petrasek, A. C.—SM
Pittman, P. D., Jr.—M
Prier, H. W.—A
Scammel, B. C.—M
Simpson, W. D.—M
Stanton, A. N.—SM
Tatum, F. W.—SM
Teasdale, A. R., Jr.—SM
Wadel, L. B.—M
Watson, J. M.—A
Weaver, S. M.—M
Wilhelm, E. S.—SM
Zeimer, D. R.—M

Denver

Allen, C. L.—S
Braunagel, M. V.—M
Capehart, M. E.—M
Case, C. W.—M
Cooper, R. R.—SGM
Davis, V. M.—SM
Donnelly, J. B.—S
Finch, M. D.—M
Hart, W. G., Jr.—M
Mielziner, Walter—A
Ostwald, L. T.—A
Spafford, L. J.—A
Stacey, D. S.—SM
Stafford, P. S.—S
Straits, R. G., Jr.—S

El Paso

Kreitler, C. E.—M
Moore, T. E.—M
Moss, E. C.—M
Rojas, A. M.—M
Schroeder, K. G.—M
Weiner, M. C.—S
Winant, A. T.—M

Ft. Worth

Allen, Rufus, Jr.—M
Askew, W. J., Jr.—M
Buehrle, C. D.—M
Condron, W. F.—A
Evans, W. L.—M
Forester, J. R.—A
Hines, R. L.—A
Jiles, C. W.—M
Muery, R. W.—M
Perry, E. R.—M
Seale, I. A.—M
Simmons, D. J.—M
Slagle, G. M., Jr.—A
Tucker, B. J.—M
Ward, E. B.—M
Watkins, O. E.—M
Webb, V. C.—M
Welter, N. E.—M

Houston

Bucy, J. F., Jr.—M
Cohn, Diter—A
Erath, L. W.—M
Giles, D. L.—S
Haill, H. K.—M
Hattz, R. F.—S
Keating, L. M.—A
Lam, C. F.—SGM
Langley, L. W.—M
Livingston, G. R.—S
Livingston, R. M.—S
McChesney, T. D.—S
Miller, K. E.—M
Navarro, S. O.—M
Patterson, H. B.—M
Richardson, A. T.—S
Sherrill, Robert—S
Smith, D. B., Jr.—A
Spence, D. W.—SM
Watson, J. K.—M
Wrye, W. C., Jr.—M

Kansas City

Brown, G. M.—S
Clarke, E. L.—A
Fayman, D. L.—M
Fox, D. N.—M
Gareis, G. E.—M
Halijak, C. A.—M
Hammond, R. C.—S
Johnson, R. E.—SGA
Miller, C. V.—M
Minnick, V. P.—S
Roehm, R. A.—S
Simonds, R. L.—M
Stout, H. L.—SM
Strevey, G. R.—S

Little Rock

Anthony, C. A.—S
Cannon, W. W.—M
Hargus, K. W.—S

Lubbock

Getman, G. A., Jr.—SGM
Griffith, P. G.—S

Jones, D. R.—S
Mills, R. C.—SGM

New Orleans

Cronvich, J. A.—SM
Dietz, W. R.—S
Drake, R. L.—M
Laurents, V. T.—A
Maus, L. C.—S
Smith, L. G.—M
Weiser, C. C., Jr.—S

Oklahoma City

Ledbetter, R. P.—M
Murta, E. J., Jr.—M
Vlay, G. J.—M

St. Louis

Adams, J. C., Jr.—S
Arndt, R. L.—S
Barr, R. K.—SGM
Bergfeld, R. F.—S
Blaine, G. J.—S
Boman, W. T., Jr.—M
Bundschuh, W. A.—M
Cassidy, R. E.—M
Crow, R. K.—M
Davis, R. T., Jr.—M
Deuschle, R. C.—S
Ferguson, L. L.—S
Fiedler, G. J.—SM
Harnisch, N. E.—SGM
Herchenreder, Herbert—M
Hieken, M. H.—M
Jones, W. S.—S
Kimmel, G. L.—S
Kortright, F. U.—A
Lago, G. V.—M
Lauher, Verlin—M
Malsbary, J. S.—M
McAninch, C. H.—M
Mayer, M. F., Jr.—M
Min, H. S.—M
Mohrman, R. F.—A
Moll, R. E.—SGM
Mundel, E. F.—SGM
Reed, D. L.—M
Sayer, J. D.—M
Scherz, C. J.—M
Schewe, R. W.—M
Schmidt, E. C., Jr.—M
Sheehan, J. S.—M
Sudfeld, C. C.—A
Taylor, Willie—M
Twombly, J. W., Jr.—M
Tucker, M. F.—M
Waters, J. I.—M
Wolf, N. F.—M
Wolin, Samuel—SM

San Antonio

Bostick, F. X.—M
Hirsch, C. O.—A
Hoffman, A. A. J.—S
King, J. D.—M
Phillips, J. P.—M
Simpson, S. H., Jr.—SM

Shreveport

Gordon, Edward—M
Long, F. V.—SM

Tulsa

Brashear, R. T.—M
Day, C. E.—M
Deveau, R. L.—S
Duerig, W. H.—A
Freeman, L. R.—M
Greening, J. P.—A
Hopkinson, E. C.—A
Labarthe, L. C.—M
Piety, R. G.—M
Rowley, R. G., Jr.—SGA
Silverman, Daniel—F
Sykora, G. E.—A
Tartar, John—S

Wichita

Oakes, J. W.—S

Region 7

Alamogordo-Holloman

Green, M. C.—SM
Guenthner, F. K.—A
Koellner, K. H.—A
Liston, D. H.—M

Albuquerque-Los Alamos

Allen, B. O.—A
Bennett, H. A.—S
Coleman, D. A.—S
Demuth, H. B.—M
Ellis, P. R., Jr.—M
Estes, S. E.—S
Fjelsted, R. P.—SGM
Hu, C. T.—S
Koschmann, A. H.—A
Koskela, A. C.—M
Linsenmayer, G. R.—M
Mohler, R. R.—M
Pace, T. L.—SM
Powell, R. L.—S
Ray, H. K.—M

Russell, J. F.—SM
Spokas, F. J.—M
Todd, J. L., Jr.—SGM
Tosti, D. T.—SGM
Warner, B. D.—M

China Lake

Creusere, M. C.—M
Dolce, S. L.—M
Dorsey, S. E.—M
Dull, M. J.—S
Foster, F. N.—M
Glatt, Benjamin—A
Kim, P. K. S.—M
Luisi, J. A.—SGM

Fort Huachuca

Hoffer, R. L.—M
Mayleben, E. F.—M
Ternow, H. G.—M

Hawaii

Witt, A. J. B.—M

Los Angeles

Abzug, M. J.—M
Akin, P. A.—M
Albon, Ralph, Jr.—A
Albrecht, Albert—SM
Allen, D. H.—SM
Alper, S. M.—M
Ambrose, J. R.—SM
Amstutz, M. F.—S
Anderson, F. A.—A
Anderson, M. J.—A
Antul, J. J.—SM
Anzel, B. M.—M
Aoki, Masanao—S
Arnold, J. R.—A
Aroyan, G. F.—M
Arsenault, W. R.—A
Aseltine, J. A.—M
Auletti, R. L.—M
Avrech, Norman—M
Aziz, R. A.—SGM
Bainbridge, B. O.—SGM
Baker, D. L.—M
Balluff, R. L.—M
Barker, A. C.—S
Barlow, D. S.—SM
Bartholomew, H. R.—M
Bartlett, F. R.—A
Beard, G. W.—M
Bekey, G. A.—M
Bekey, Ivan—A
Belding, R. A.—SGM
Bement, W. A.—M
Bemis, R. C. B.—A
Benning, F. N.—M
Bernstein, P. P.—SM
Berry, R. P.—M
Bertorello, R. J.—M
Bills, G. W.—M
Blum, L. M.—M
Bold, N. T.—S
Bonney, R. B.—SM
Booton, R. C.—M
Borgeson, P. W.—M
Borsch, K. S.—SM
Bower, J. L.—SM
Boxer, Rubin—M
Boykin, T. R., Jr.—M
Braun, E. L.—SM
Brewer, H. E.—M
Bricken, G. I.—S
Broadwell, W. B.—M
Brock, P. A.—M
Bronstein, L. M.—A
Bunge, W. R.—A
Burk, W. A.—M
Burland, R. N.—M
Burnsweig, Joseph, Jr.—M
Busse, C. A.—A
Buxher, F. X.—SM
Byck, D. M.—A
Cahn, J. M.—A
Callot, Sherman—A
Campbell, Graham, Jr.—M
Canova, G. M.—M
Capps, J. W.—M
Carlson, C. O.—A
Carmel, H. H.—M
Chamberlain, A. T.—M
Chamorro, R. D.—M
Chandler, D. P.—M
Chang, Bansun—M
Chang, Y. N.—M
Chapman, C. W.—SM
Chapsky, Jacob—M
Chase, H. E.—M
Cheng, F. H. A.—SGM
Cherniack, A. E.—SGM
Chestnut, M. G.—A
Christensen, A. V.—M
Clark, P. A.—M
Cochran, E. D.—M
Cohill, G. I.—SGM
Collins, D. H.—A
Collins, R. F.—M
Conway, J. P.—S
Cornell, J. R.—M
Cosgrave, S. J.—M
Cowart, B. E.—M
Cox, J. A.—M
Craven, W. A., Jr.—SM

Cribbs, W. J.—SGM
Cromleigh, R. G.—A
Crooke, V. E.—SM
Crowe, J. W.—M
Curl, G. W.—A
Cutting, Elliott—A
Dautremont, J. L., Jr.—M
Davidow, W. H.—S
Davidson, R. G.—S
Davis, H. G., Jr.—M
Dawirs, N. L.—S
Deaux, F. J.—M
Deen, J. R.—M
Deming, A. F.—M
Denton, R. F., Jr.—M
Deuser, D. A.—M
Diemer, F. P.—M
Doty, R. L.—SM
Drucker, A. N.—M
Dzilvelis, A. A.—M
Easterling, M. F.—M
Edelsohn, C. R.—M
Eggeman, D. J.—M
Egger, Alexander—A
Eichwald, W. F.—M
Eikelman, J. A., Jr.—M
Eisner, William—M
Ellis, D. O.—M
Endo, F. Y.—M
Engel, H. L.—A
Eno, R. F.—M
Epstein, Saul—A
Erway, D. D.—SGM
Eschner, Albert, Jr.—SM
Estrin, Gerald—M
Fatton, G. A.—A
Ficher, R. C.—SGM
Finley, W. A.—A
Fischer, P. F.—SM
Fish, K. L.—A
Fish, W. Y.—M
Fisher, H. V., Jr.—M
Fitzgerald, G. E.—M
Fleeman, P. J.—M
Floyd, G. W.—A
Fortier, R. E.—M
Foster, W. H.—M
Fox, A. J.—S
Fox, C. W.—M
Foxman, Eugene—A
Frady, W. E.—M
Friedenthal, M. J.—APG
Fuhrman, T. A.—A
Fujimoto, Yoshiaki—M
Fursa, Alex—M
Gabler, R. T.—SM
Garber, L. F.—M
Gardner, F. M.—A
Gardner, F. H.—SM
Gauronskas, P. P.—M
Gaylord, R. S.—M
Gebhardt, C. C.—M
Gee, L. C.—M
George, R. D.—M
Giese, Clarence—M
Gilbert, J. D.—M
Gillis, D. E.—M
Glowalla, John—M
Goldstein, Albert—M
Goodwin, J. W.—A
Gottmer, G. W.—M
Grabbe, E. M.—A
Graham, W. R., Jr.—S
Greenberg, E. L.—M
Griffith, S. D.—M
Groves, C. R.—S
Gruber, Kenneth—SGM
Gullatt, S. P., Jr.—A
Gunning, W. F.—M
Gustin, J. T.—A
Hailey, R. D.—A
Haines, W. K.—S
Hall, C. R.—M
Hall, G. D.—A
Hanna, R. E.—M
Hansen, G. J.—S
Hanson, M. E.—M
Hara, H. H.—S
Harmon, W. G.—A
Hartmann, Sigmund—M
Hawkins, J. K.—M
Hawley, A. E.—M
Hayes, J. E.—M
Hedges, C. P.—SM
Heikkinen, R. R.—M
Helgeson, B. P.—A
Hendrick, James, Jr.—M
Heyliger, G. E.—M
Heyning, J. M.—M
Hill, R. E.—M
Hinrichs, Karl—SM
Hitchcock, R. W.—M
Hobbs, G. H.—M
Horwitz, L. B.—SM
Hoskinson, E. A.—M
Howard, D. R.—SGM
Hoy, E. C.—M
Hoyt, M. W.—M
Hruby, R. J.—M
Hughes, C. J.—M
Hunt, E. B.—M
Hutcheon, R. S.—SM
Hutton, L. G.—S
Imhoff, J. J.—SM
Izuel, A. G.—SGM

Jack, R. W.—M
Jackson, K. R.—SM
Jacobson, O. M.—M
Jeschke, A. W.—A
Joerger, J. C.—M
Johnson, J. S.—M
Johnson, M. G.—S
Johnson, R. W.—SM
Johnson, W. A.—M
Jones, R. W.—M
Judge, F. W.—M
Justice, L. E.—M
Kahn, B. S.—M
Kalayjian, John, Jr.—M
Kaplan, L. M.—M
Karr, P. R.—SM
Kasten, C. L.—M
Kaufman, F. H.—M
Kaufman, Sidney—M
Kawahata, B. I.—M
Kawana, H. Y.—A
Kazarian, D. G.—M
Kennan, J. E.—A
Kennedy, C. J.—SM
Kennedy, F. D.—M
Kenny, P. C.—M
Keppel, R. A.—SM
Kercher, R. B.—M
Kern, W. W.—A
Kerster, George—M
Kiesel, R. H.—M
King, C. G., Jr.—M
King, Edward—M
King, J. E.—A
Kirk, C. N.—M
Kirsch, H. A.—M
Kishi, F. H.—A
Kitabayashi, Tamotsu—A
Klein, E. A.—M
Knepper, R. C.—M
Knox, R. V.—M
Krill, C. K.—SM
Kroy, W. H., Jr.—M
Kuerschner, Helmut—M
Kukel, Joseph—M
Kuntz, G. L.—A
Lachenmeier, G. E.—A
Landyshev, Alexander—SM
Lawrence, A. F., III—M
Lebell, Don—M
Lee, D. H.—S
Lee, H. J.—A
Lees, A. B.—M
Leon, H. I.—SM
Leondes, C. T.—SM
Leone, W. C.—A
Lessley, T. D.—M
Levine, Leon—A
Levine, S. E.—A
Levinson, R. M.—A
Levy, E. C.—M
Levy, L. S.—M
Lieberman, A. G.—M
Lilienstein, Manfred—SM
Lillibridge, E. H.—A
Lindberg, H. E.—M
Lindholm, C. R.—M
Linvill, W. K.—M
Lou, C. B.—M
Loughran, G. P.—S
Louie, William—A
Lowrance, L. W.—A
Lubeck, R. V.—SGM
Luck, Robert—M
Lyons, L. H.—M
Mack, P. H.—A
Mancini, A. R.—M
Manly, Ron—A
Manner, Horace—M
Margolis, Jack—M
Margolis, Maier—A
Marshall, Bill—M
Marshall, R. L.—M
Martin, Devereaux—SM
Martin, T. E.—A
Masenten, W. K.—S
Mason, P. V.—S
Mayberry, L. A.—SM
Mayner, G. S.—A
McCarthy, J. W.—M
McComb, B. J.—M
McCormick, G. F.—M
McFadden, B. W.—M
McGann, Laurence—S
McGhee, R. B.—M
McKeever, B. T.—S
McLarin, Maitland—M
McLeod, M. G.—M
McRuer, D. T.—SM
Mehner, E. W.—A
Meissinger, H. F.—SM
Merrill, H. M.—M
Metzner, H. E.—M
Migliaccio, Albert—M
Miller, C. E.—SM
Miller, D. S.—A
Miller, R. A.—M
Mitsutomi, Takashi—A
Modlinski, W. M.—M
Moon, W. D.—SGM
Moore, M. E.—M
Morell, C. S.—M
Morgan, H. C.—A
Morrison, A. I.—M
Mortensen, R. E.—S

Morton, W. B., Jr.—M
Mosbrooker, M. L.—S
Mosher, W. W., Jr.—S
Moss, J. F., Jr.—A
Mullin, F. J.—SGM
Mundt, Edward—M
Nakano, Hiroshi—M
Neal, C. B.—S
Nedland, E. H.—A
Neeley, J. C.—M
Nelson, C. S., Jr.—M
Newman, D. B.—SM
Nickerson, H. H.—A
Niemoeller, D. E.—S
Noda, Mitsuaki—M
Notham, M. H.—M
Nuban, Ebrahim—M
Nussbaum, O. N.—A
Nuttall, H. V.—SM
Oehlke, A. E.—M
O'Hara, C. L.—A
Ohrenstein, S. B.—A
Okimoto, J. Y.—M
Olsen, J. D.—S
Olsen, L. V.—M
Olson, C. P., Jr.—M
Olson, N. L.—A
O'Neil, W. F.—M
Padgett, E. P., Jr.—A
Padwa, M. N.—A
Paquin, J. H.—SM
Parker, A. T.—M
Parzl, R. C.—M
Patrusky, Nathan—SM
Pekrul, P. J.—M
Perez, A. A.—M
Peringer, Paul—A
Perkins, L. M.—M
Pernick, L. J.—M
Peterson, E. J.—M
Peterson, R. W.—M
Phelps, J. F., Jr.—M
Phister, Montgomery, Jr.—A
Piereson, R. G.—M
Pierson, D. N.—SGM
Pinson, E. N.—S
Pitman, G. R., Jr.—M
Pixley, N. S., Jr.—M
Plank, Charles—M
Plotkin, S. C.—M
Poirier, J. P. A.—S
Post, Geoffrey—M
Poulson, W. A.—M
Pratt, S. L.—M
Primack, Jerry—S
Primozich, F. G.—A
Pullen, E. W.—S
Qua, W. C.—A
Quackenbush, R. E.—A
Quinn, T. B.—A
Radant, M. E.—M
Raffensperger, M. J.—SM
Ramstedt, C. F.—M
Rau, J. M., Jr.—M
Redden, E. T.—A
Redmond, J. G.—M
Rehler, K. M.—M
Reich, R. F.—M
Rescoe, J. M.—SM
Reynolds, C. M., Jr.—S
Rickords, T. J.—M
Rieman, F. C.—M
Robinson, J. A.—M
Robinson, L. M.—M
Robinson, L. P.—SM
Rockey, R. J.—M
Roe, G. G., Jr.—M
Rogers, J. G.—A
Rogers, J. M.—A
Rogers, T. A.—M
Rosenberg, R. E.—A
Rosenbloom, F. H.—SGM
Ross, D. H.—M
Ross, Irving—A
Rothe, C. W.—M
Ruiz, M. L.—M
Russell, W. J.—M
Russell, W. T.—SM
Sabol, R. W.—SM
Sakai, R. V.—SGM
Salmi, T. W.—M
Salzer, J. M.—M
Sanderson, K. W.—SGM
Sandoval, K. A.—M
Sarture, C. W.—M
Savo, T. A.—M
Sawyer, H. F.—M
Sayano, K. F.—M
Schalk, Norbert—M
Schandl, Emil—M
Scharer, M. E.—M
Schiowitz, H. H.—A
Schmidt, L. O.—S
Schneider, R. L.—M
Schneider, Stanley—A
Schnoor, J. E.—A
Schroeder, R. L.—A
Schroeder, William—M
Schull, G. R.—M
Schulte, R. W.—A
Schultz, P. R.—M
Schultz, R. T.—A
Schulz, K. S.—M
Scope, Sol—M
Scott, R. G.—M

Seelig, Leopold—S
Seitz, F. R.—VA
Seitz, S. S.—M
Sensiper, Samuel—SM
Shearer, G. W.—M
Shelley, R. G.—SM
Shimizu, Kaoru—M
Shoop, W. F.—M
Shuler, M. H.—M
Shutt, S. G.—A
Siegel, J. C.—M
Silva, L. M.—M
Simkins, E. D.—M
Simon, H. S.—M
Sizemore, L. E.—M
Skeirik, R. M.—M
Slocomb, G. M.—M
Small, R. H.—SGM
Smiley, E. L.—M
Smith, B. N.—A
Smith, J. D.—SGM
Smith, L. C.—A
Smith, R. E.—M
Smith, R. A.—M
Smith, W. B.—M
Smokler, M. I.—M
Smyth, R. K.—M
Snapp, K. M.—A
Sofen, I. A.—A
Sohler, J. F.—M
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